An information-theoretic imaging approach to pose search for transmission tomography*

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Abstract

Using a nonlinear model that accounts for the Poisson data acquired in transmission tomography, as well as effects such as noise, scatter, and spectral hardening, we developed an alternating minimization algorithm for image reconstruction. The algorithm is adapted to include a search for the pose of known objects in the reconstructed image, thus improving the image quality and the convergence rate.

Computerized tomographic (CT) imaging of soft tissue containing high-density objects, such as surgical clips and metal rods, has been a widely studied subject. A recent application has been to use CT for nondestructive testing of objects such as automotive or aeronautical components and ceramics. Conventional image reconstruction algorithms, such as filtered backprojection, are based on a linear model and assume that the set of CT measurements is the Radon transform of the linear attenuation coefficients of the scanned object. This assumption is quite harmless when the object's components have similar values, but when high-density materials are present, the failure to consider the Poisson nature of the data and the nonlinear effects of noise, spectral hardening, scatter, and energy dependence of the coefficients leads to images with severe streaking artifacts.

Thus, our goal is to form a suitable model for transmission tomography that accounts for the nonlinearities, form a reconstruction algorithm based on that model, and then incorporate into the algorithm some prior knowledge of the object being scanned, such as a knowledge of the high-density objects. We index the pixels by the set, X, and the source-detector pairs by the set, Y. Let $h(y|x)$ be a nonnegative kernel determined by the scanner geometry, and define it as the average path length through the image pixel x on the path defined by the source-detector pair y. Let $I_0(y,E)$ be the mean number of incident photons received at a detector over a spectrum of energies, E, in the absence of attenuating objects. We assume for our model that the detectors are photon-counting devices, so that the measured data, $d(y)$, are Poisson-distributed, with means:

$$g(y) = \sum_E I_0(y,E)\exp\{-\sum_{x\in X}\sum_i h(y|x)\mu_i(E)c_i(x)\} + \beta(y), \tag{1}$$

where $\mu_i(E)$ is the attenuation spectrum for the ith constituent (e.g. water, bone, metal, ceramics), and $c_i(x)$ is the specific gravity function that we wish to reconstruct. The sum

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in the exponent (the forward projection) is a discrete approximation of the line integral of attenuation along each source-detector path, and the exponential term is the survival probability of the photons traveling on that path. The final term in the expression, $\beta(y)$, is the mean of the background events and it is included here to model photon scatter.

To form the image estimate, we want to maximize the likelihood of $d(y)$ over $g(y)$ of the form (1), or equivalently, minimize the I-divergence $I(d\|g) \equiv \sum_{y \in Y} d(y) \ln[d(y)/g(y)] - d(y) + g(y)$. Building on the methods of Lange and Carson [1] for transmission tomography and using constrained minimization techniques, we rewrite this as a double minimization over p and q [2], where p belongs to a linear family that enforces the constraints imposed by the data $d(y)$, and q belongs to an exponential family such that q is a mean of the data for a given energy. The resulting alternating minimization (AM) iterative algorithm alternates between finding p and q, and we have shown the iterations to be as follows:

$$\hat{c}_i^{(k+1)}(x) = \hat{c}_i^{(k)}(x) - Z_i(x)^{-1} \ln\left(\tilde{b}_i^{(k)}(x)/\hat{b}_i^{(k)}(x)\right), \tag{2}$$

where $\tilde{b}_i^{(k)}(x)$ and $\hat{b}_i^{(k)}(x)$ are *backprojections* of the kth estimates of p and q, respectively, and $Z_i(x)$ is an appropriate scaling function.

In a simple experiment using noiseless, synthetic data (containing high-density material), convergence of the image was very slow. If we assume full knowledge of the position and orientation (collectively called the *pose*) and composition of the high-density object(s), however, we can constrain these pixel values to have the known attenuation coefficients and the algorithm over the remainder of the image converges very quickly.

Of course, it is not realistic to think that we know, *a priori*, the precise pose of a known object, but this test gives us hope that we can use the prior knowledge about these high-density objects to obtain much faster convergence. By translating and rotating a reference image of the object by some proposed pose θ, we have a known function $c_a(x:\theta)$ that is mostly zeros, except for the relatively few pixels that are covered by the known objects. We rederived the alternating minimization algorithm to simultaneously search for θ while reconstructing the image by including the following constraint: *every pixel in a candidate image for iteration k must be greater than or equal to $c_a(x:\hat{\theta}^{(k)})$.* This not only enforces a nonnegativity constraint on our image but also allows us to set the pixels that are entirely covered to their exact value.

The algorithm goes as follows: 1) Define $c^{AM}(x)$ to be the expression in (2). 2) Pick a candidate pose θ. If the pixel is completely covered by the object, then $\hat{c}^{(k+1)}(x:\theta) = c_a(x:\theta)$. If not, then $\hat{c}^{(k+1)}(x:\theta) = \max[c^{AM}(x), c_a(x:\theta)]$. 3) Forward project $\hat{c}^{(k+1)}(x:\theta)$, form $g(y)$ by (1), and then find $I(d\|g)$. 4) Try a suitable number of candidate θ values in a lattice of smaller and smaller stepsizes surrounding the current pose, and select the pose and corresponding image yielding the smallest I-divergence.

Results have shown that we can accurately obtain the correct pose, and the streaking artifacts have been dramatically smoothed. Further investigation of regularization approaches is needed, however, to achieve appropriate smoothing, especially at edges.

References

[1] K. Lange and R. Carson, "EM reconstruction algorithms for emission and transmission tomography," *J. Comp. Assisted Tomo.*, vol. 8, pp. 306–316, Apr. 1984.

[2] J. A. O'Sullivan, "Alternating minimization algorithms: from Blahut-Arimoto to expectation-maximization," in: *Codes, Curves, and Signals—Common Threads in Communications* (A. Vardy, ed.), Kluwer Academic Publishers, pp. 173–192, 1998.