Information Theoretic Imaging

WU Faculty: J. A. O'Sullivan

WU Doctoral Student: Naveen Singla

Boeing Engineer: James Meany

First Year Focus:

Imaging for Data Storage

Image Reconstruction → Data Retrieval

Washington University in St. Louis
SCHOOL OF ENGINEERING & APPLIED SCIENCE

Information Theoretic Imaging

- Theoretical and Implementation Issues in
  - Image formation
  - Image deblurring
  - Automatic target recognition
  - Data modeling
- Data Storage Systems
  - Channel coding problems
  - Graphical models for available data
  - Binary rather than continuous data
- Cross-Fertilization
  - Imaging approach to data storage
  - Graphical models for image formation, ATR

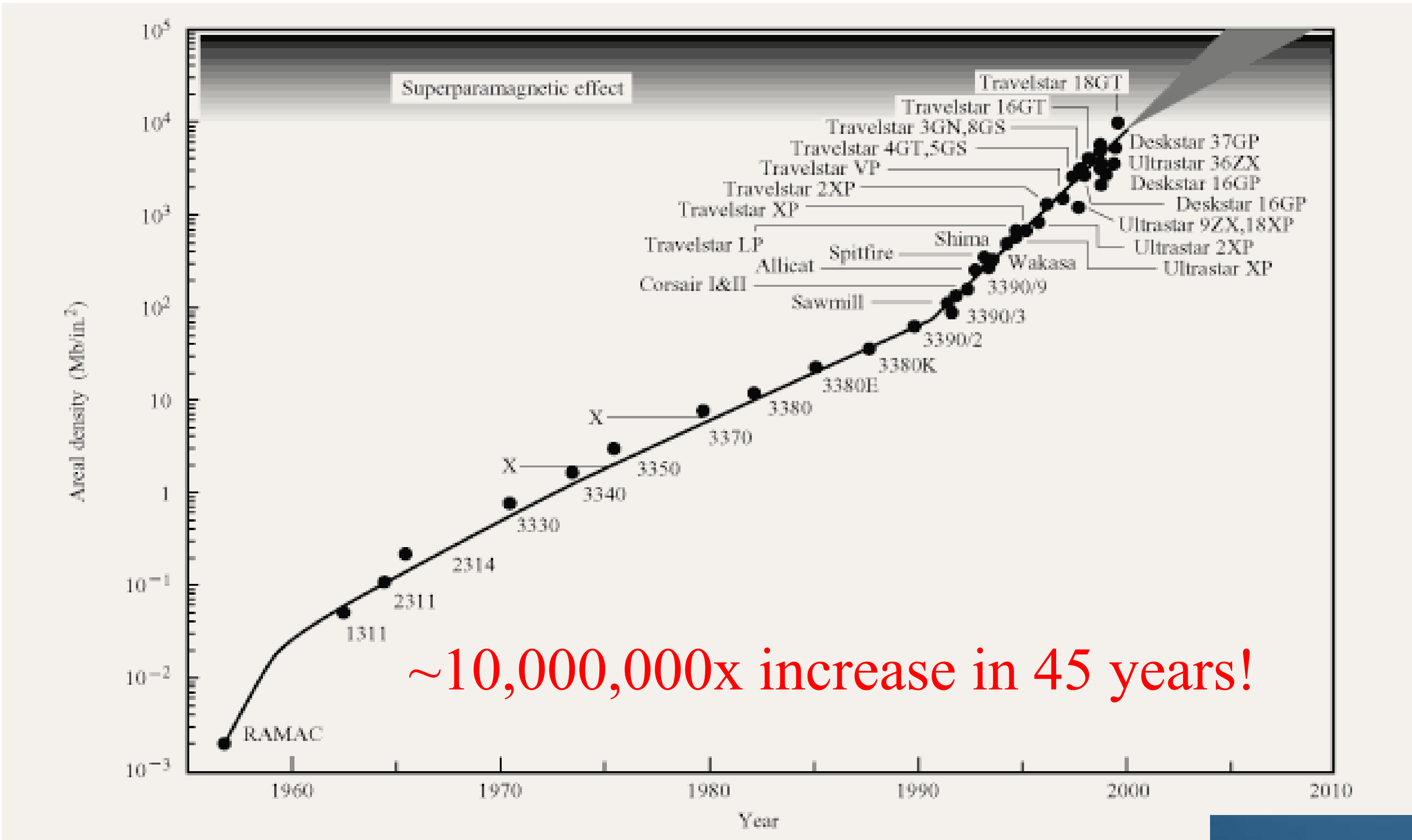
Joint Detection and Decoding Schemes for 2D Recording

J. A. O'Sullivan, N. Singla,
Y. Wu, R. S. Indeck

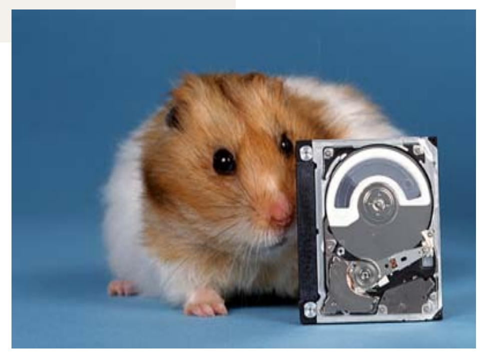
Electronics Systems and Signals Research Laboratory
Magnetic Information Science Center
Washington University

Washington University in St. Louis
SCHOOL OF ENGINEERING & APPLIED SCIENCE

Enabling Technology: Disk Drives

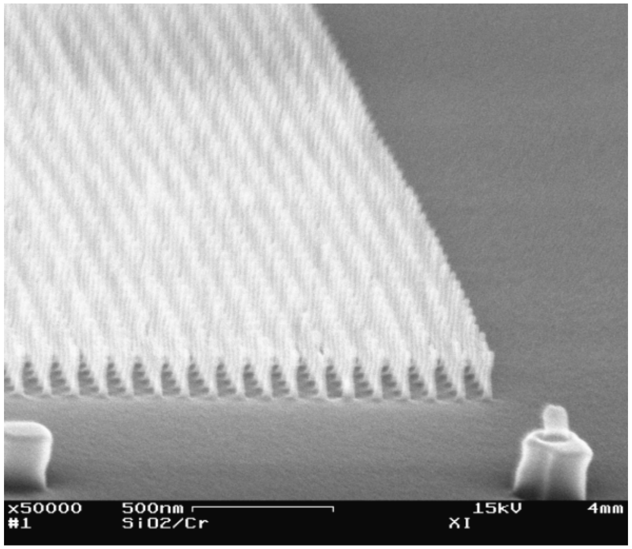


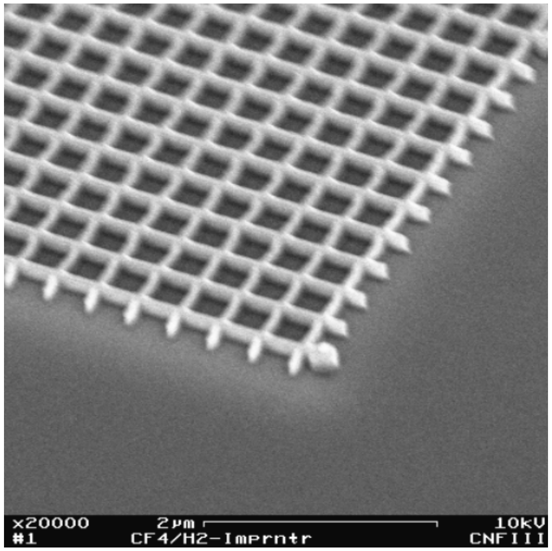
Magnetic disk storage areal density
vs. year of IBM product introduction
(From D. A. Thompson)



Problem Description

- Reliable data retrieval from channels having 2-D ISI
  - Advanced storage media: Patterned media
  - As bit aspect ratio reduces inter-track interference becomes significant
  - Optical memories

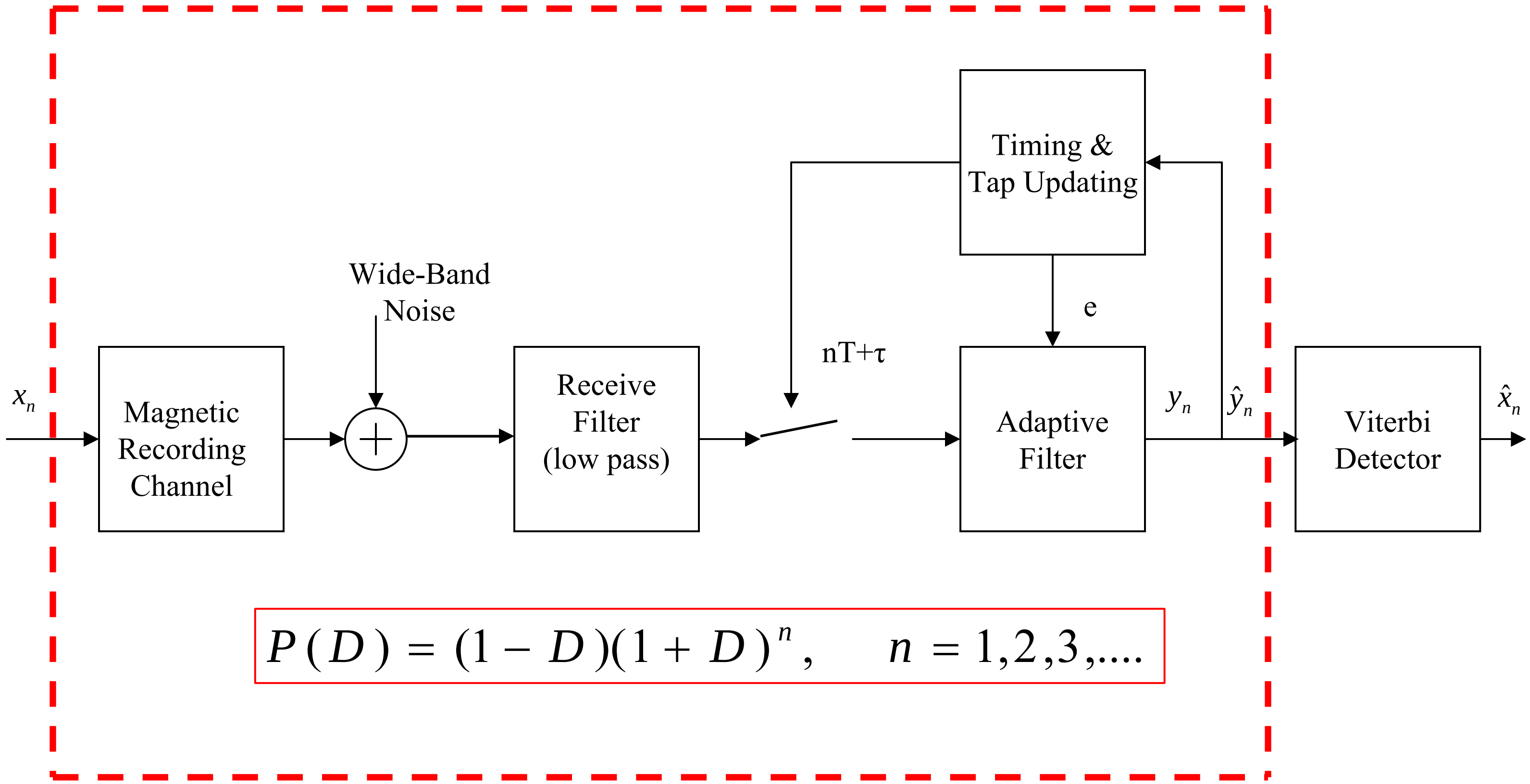




Science enables:

6 million-million bits/square inch!

Conventional PRML System



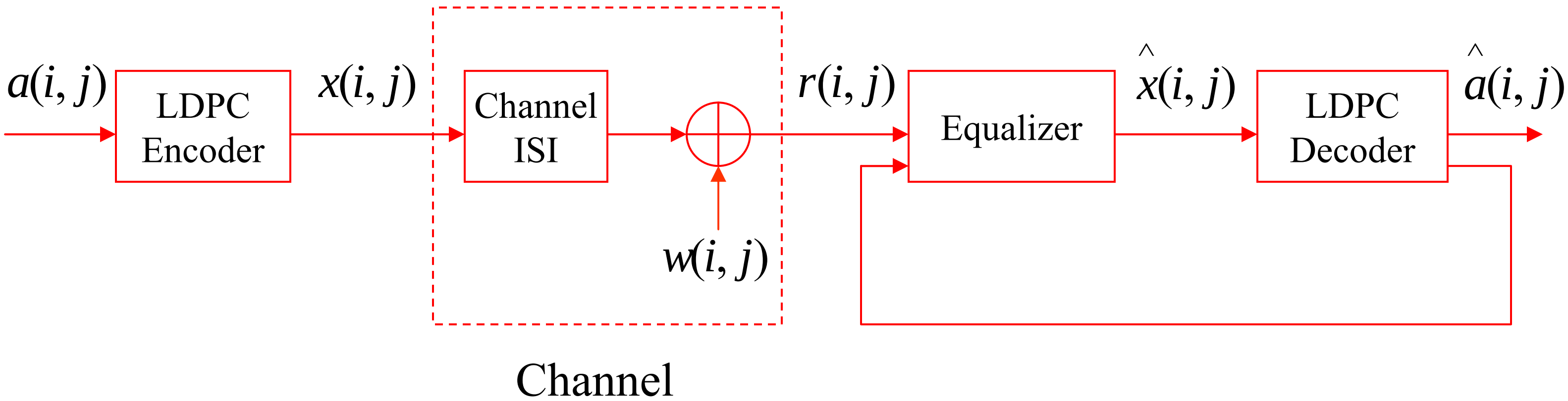
No generalization to 2D

Joint Equalization and Decoding Schemes

- General 2D ISI
  - Using 2D MMSE equalization and decoding
  - Using novel message-passing algorithms that take advantage of the 2D dependence
- Separable 2D ISI
  - Using turbo equalization

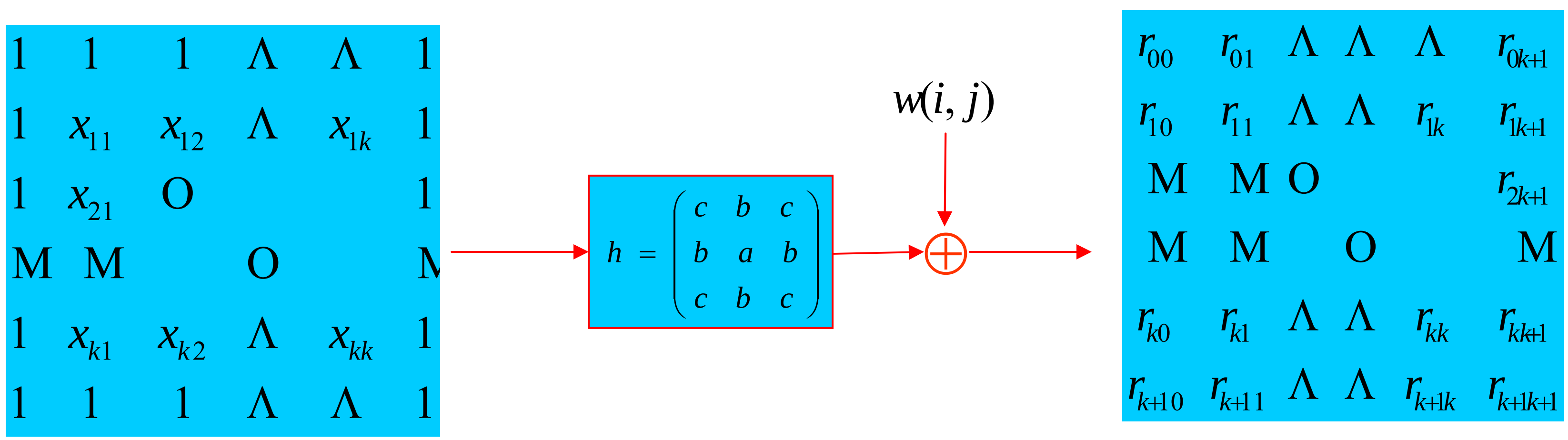
Singla *et al.*, "Iterative decoding and equalization for 2-D recording channels," *IEEE Trans. Magn.*, Sept. 2002.

Channel Model



- $x(i,j) \in \{+1,-1\}$
- Channel ISI is 2D and linear
- Noise assumed to be AWGN

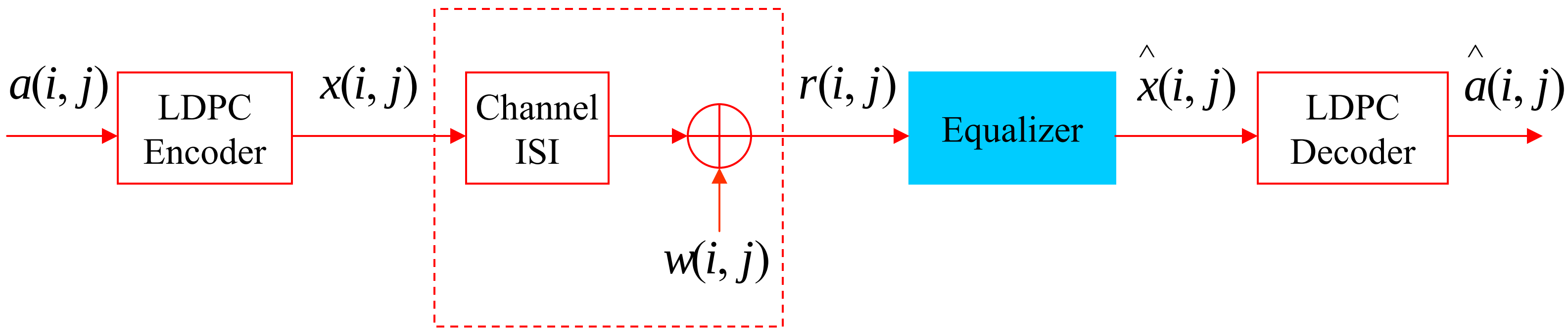
2D Intersymbol Interference



GUARD BAND

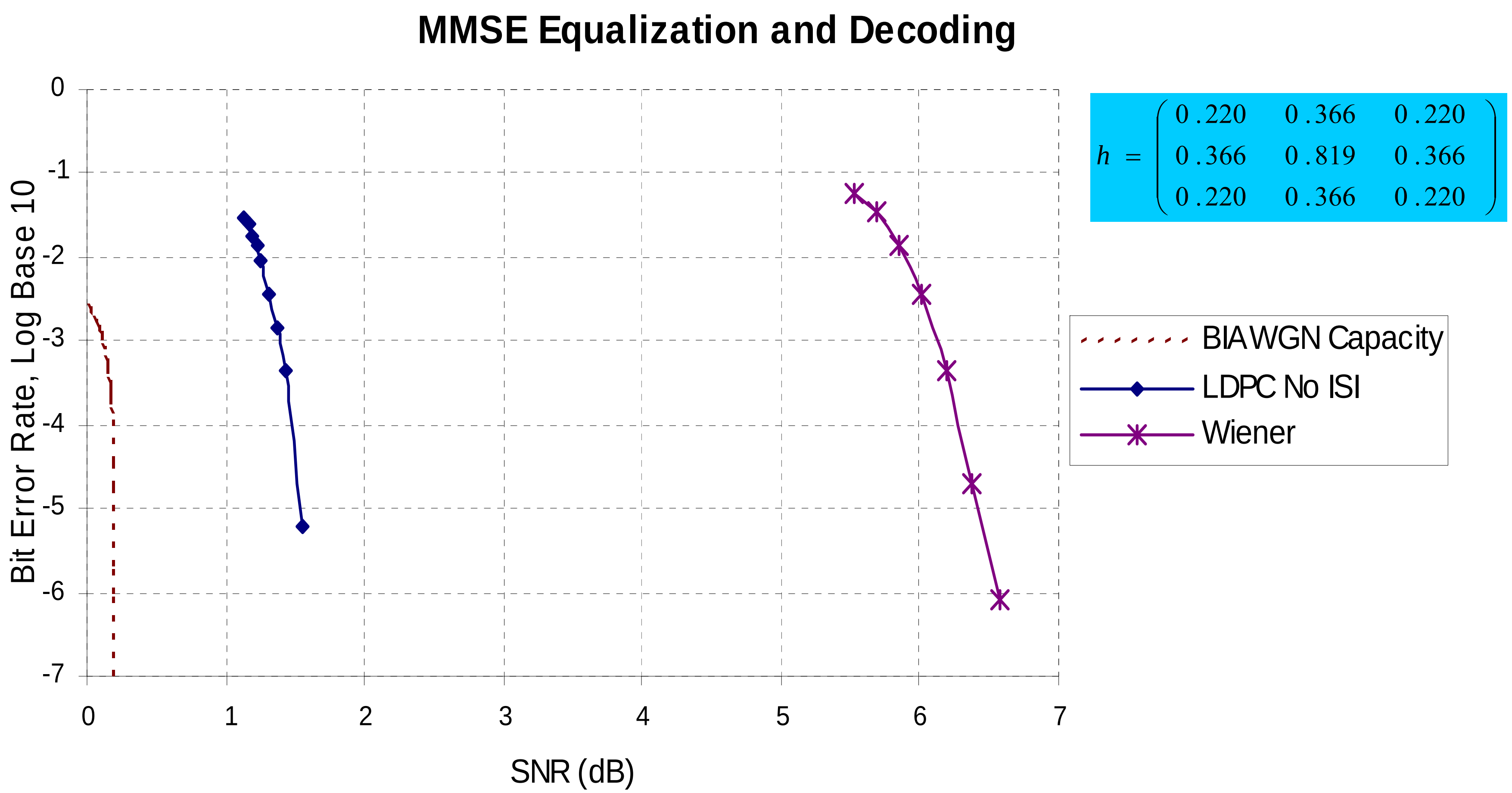
$$r(i,j) = \sum_{m,n=0}^{2} x(i-m, j-n)h(m,n) + w(i,j)$$

MMSE Equalization and Decoding

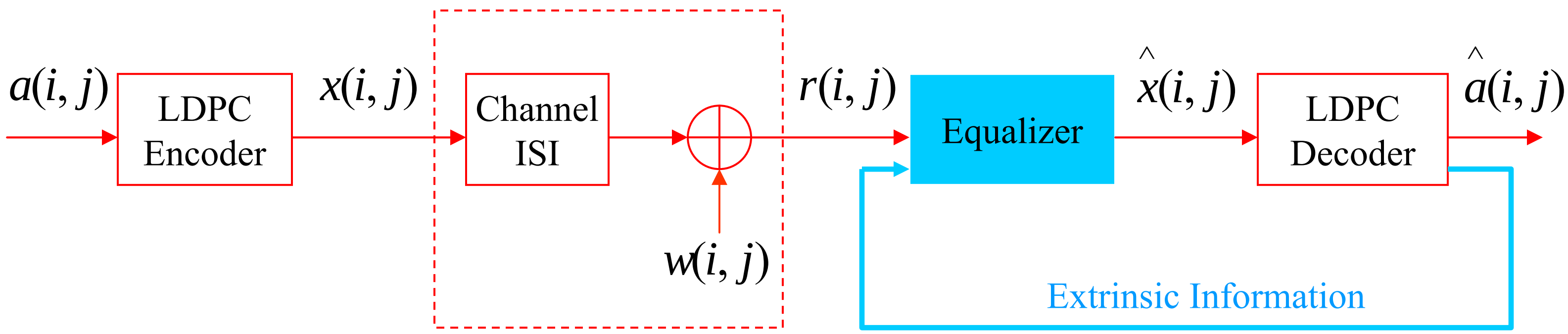


- Equalizer designed assuming inputs to be Gaussian

Performance



Iterative MMSE Equalization and Decoding

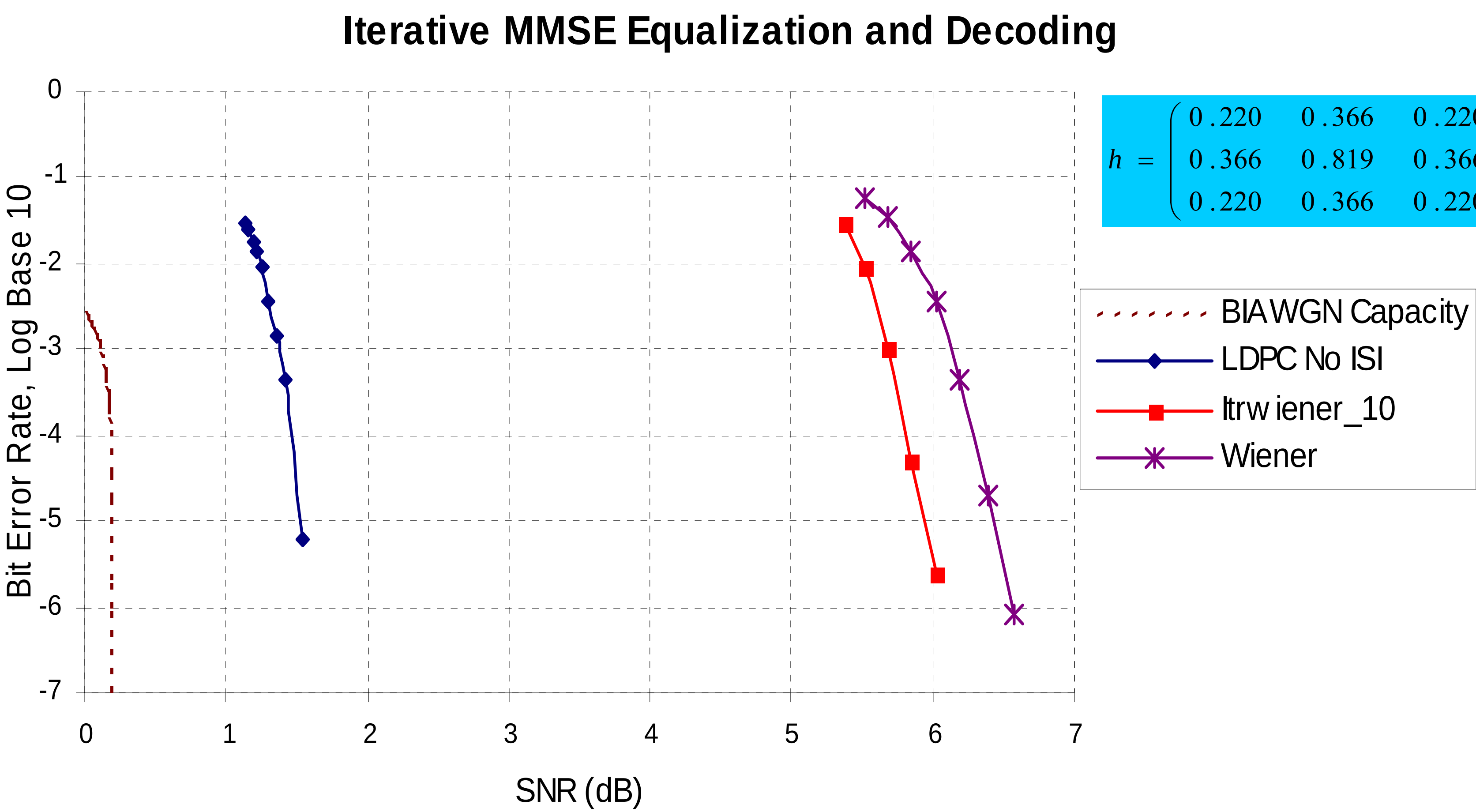


- Soft information, estimated mean of the codeword, passed from LDPC decoder to equalizer

$$E[x] = \Pr(x=1) - \Pr(x=-1)$$

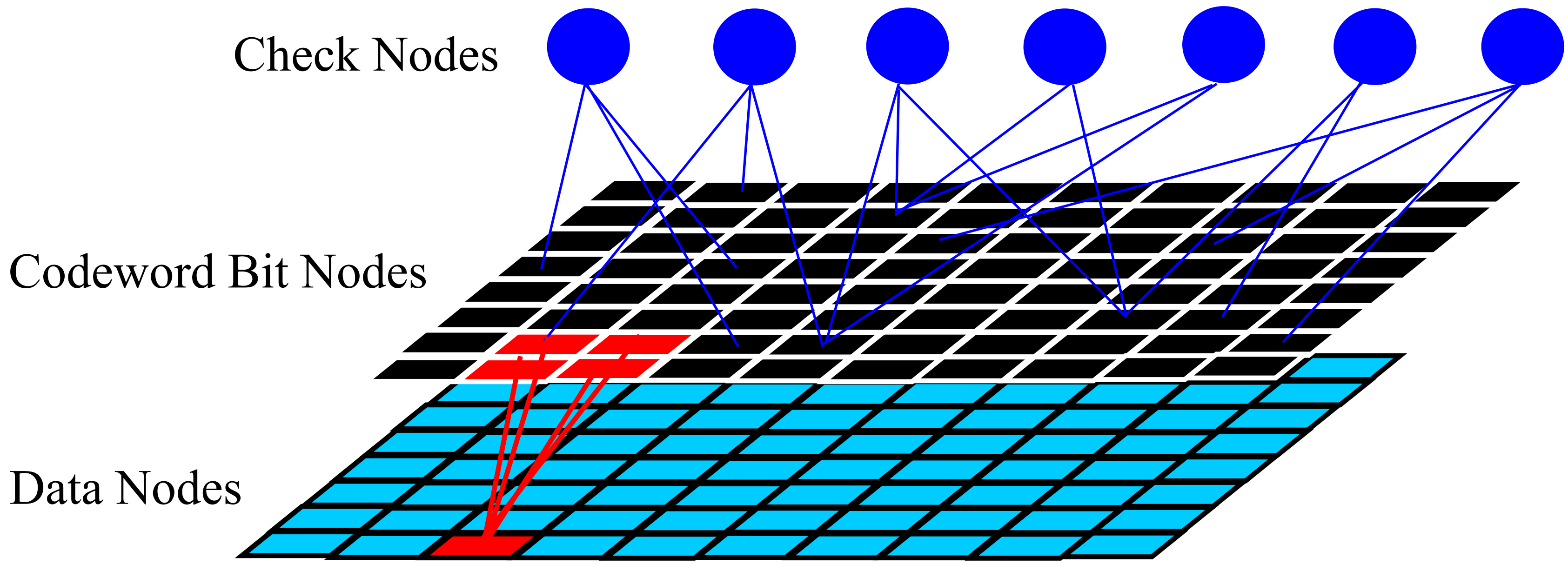
$$\hat{x} = E[x] + W**[r - h**E[x]]$$

Performance



$$h = \begin{pmatrix} 0.220 & 0.366 & 0.220 \\ 0.366 & 0.819 & 0.366 \\ 0.220 & 0.366 & 0.220 \end{pmatrix}$$

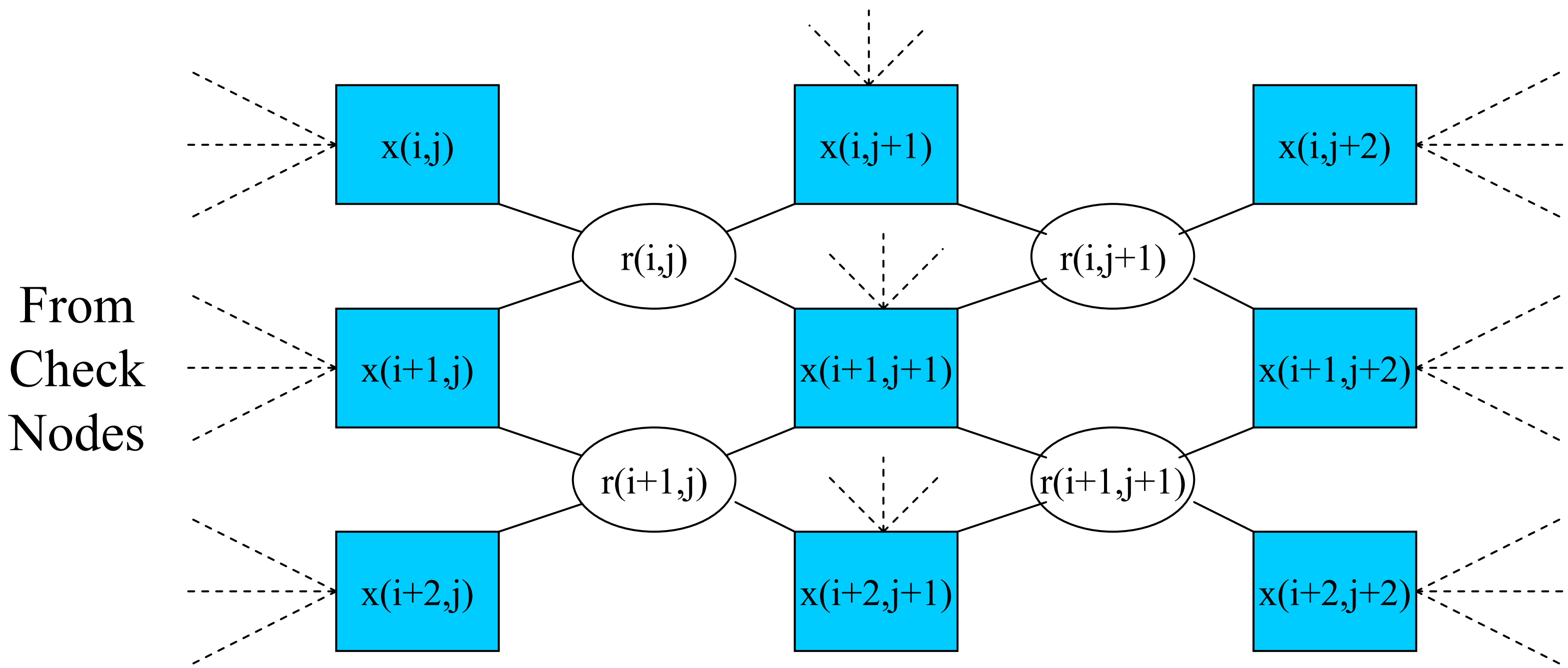
Full Graph Message-Passing



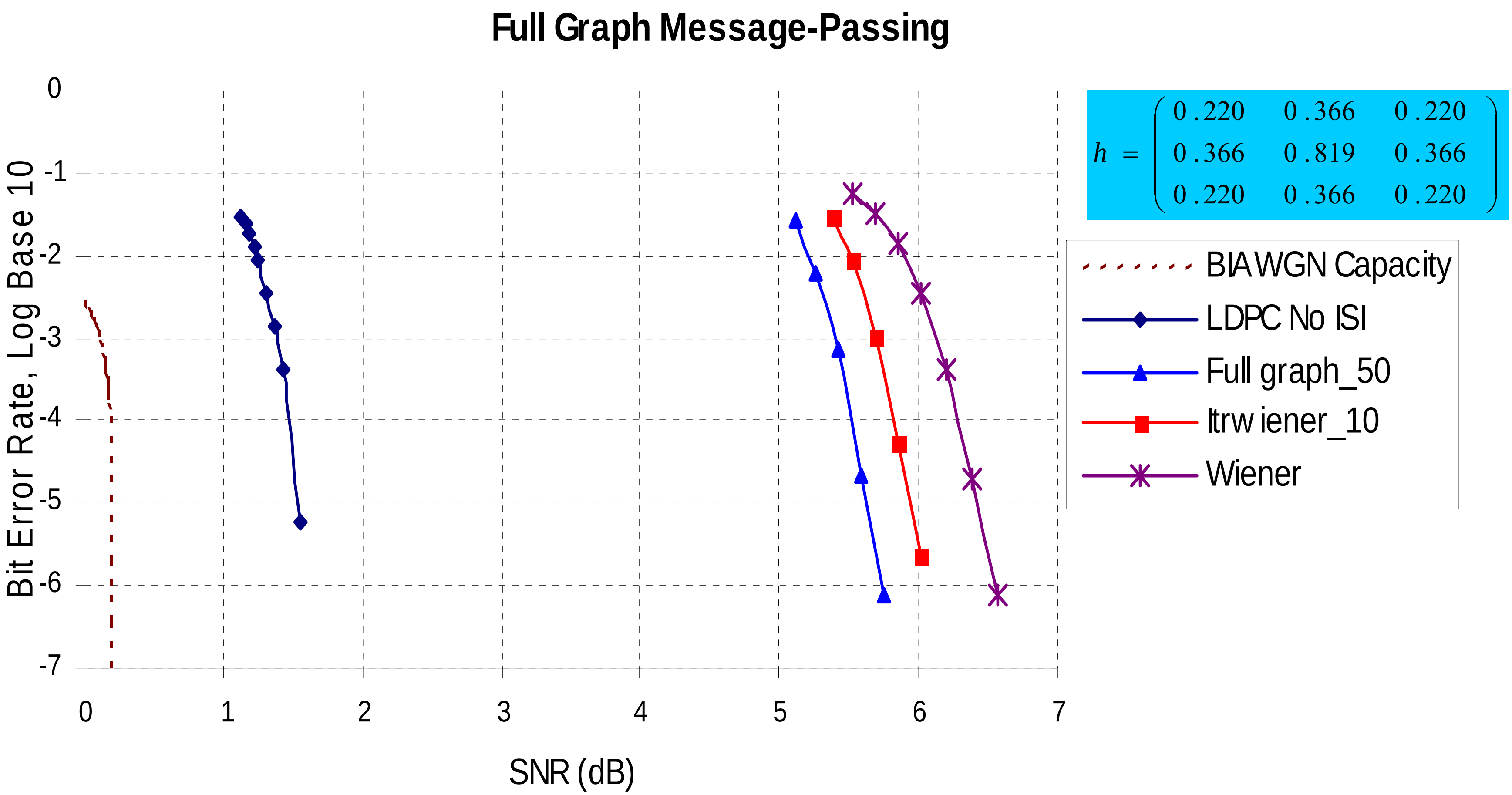
$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix}$$

$$r_{i,j} = x_{i,j} + 0.5x_{i-1,j} + 0.5x_{i,j-1} + 0.25x_{i-1,j-1} + w_{i,j}$$

Full Graph


From
Check
Nodes
x(i,j)
x(i,j+1)
x(i,j+2)
r(i,j)
r(i,j+1)
x(i+1,j)
x(i+1,j+1)
x(i+1,j+2)
r(i+1,j)
r(i+1,j+1)
x(i+2,j)
x(i+2,j+1)
x(i+2,j+2)

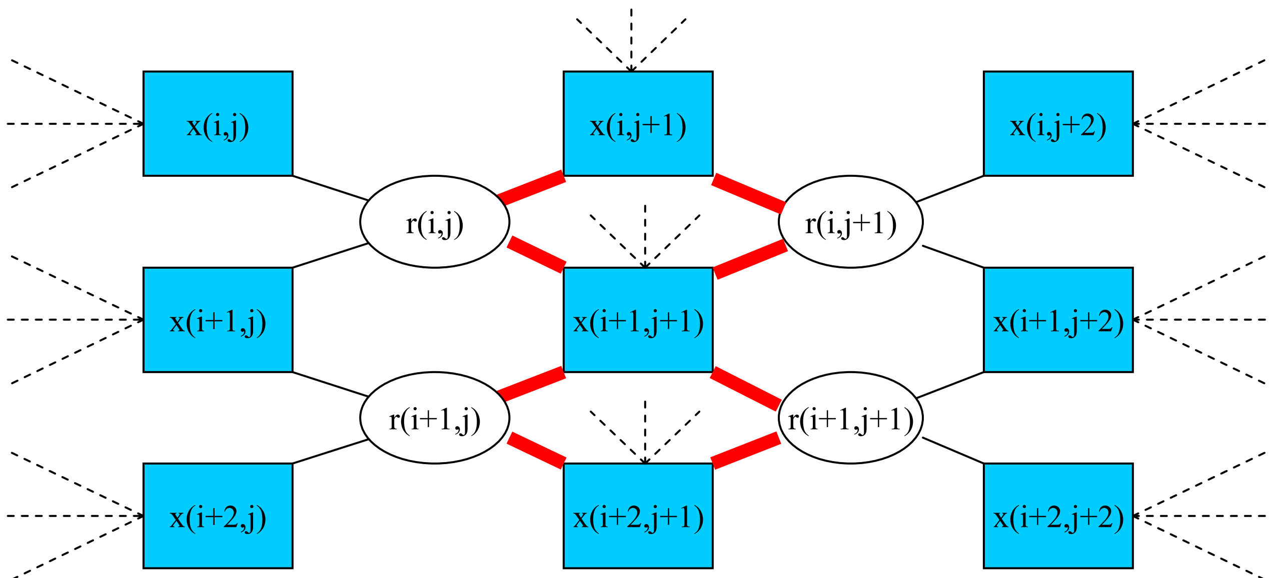
Performance



Full Graph Analysis

- Length 4 cycles present which degrade performance of message-passing algorithm

From Check Nodes



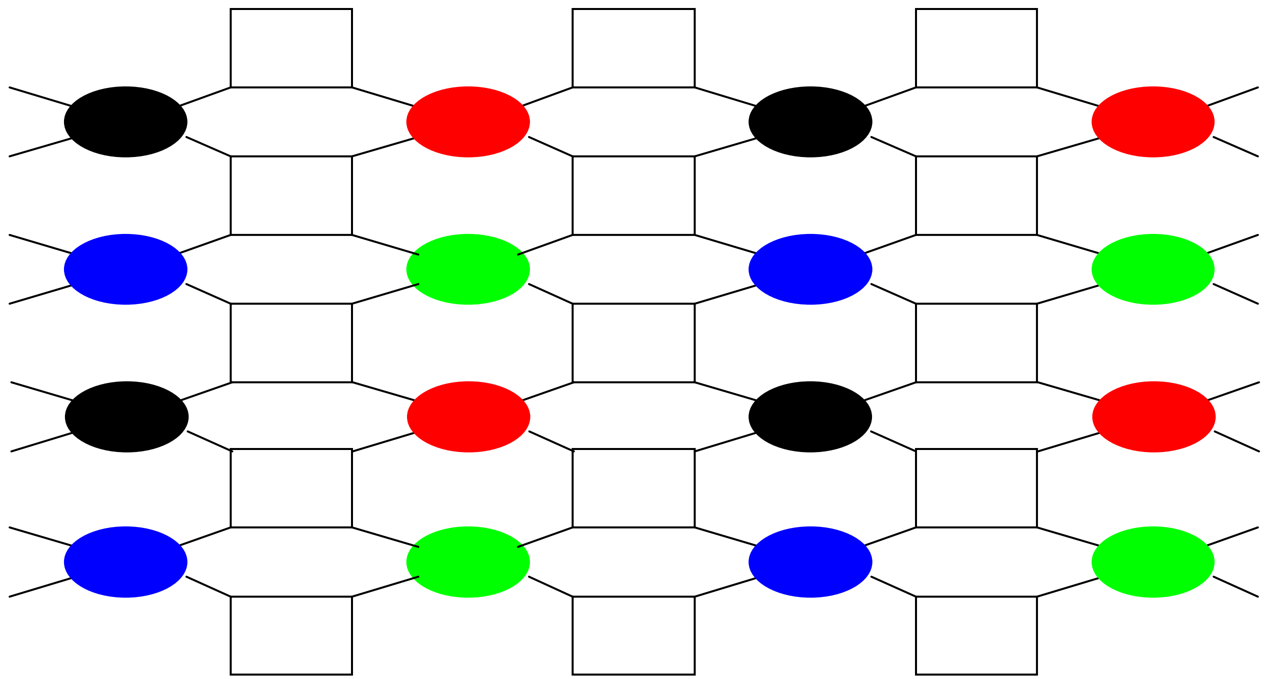
Kschischang *et al*., "Factor graphs and the sum-product algorithm," *IEEE Trans. Inform. Theory*, Feb. 2001.

Ordered Subsets Message-Passing

- From Imaging – Data set is grouped into subsets to increase rate of convergence
- For Decoding – Observed data is grouped into subsets to eliminate short length cycles in the Channel ISI graph

H. M. Hudson, and R. S. Larkin, "Accelerated image reconstruction using ordered subsets of projection data," *IEEE Trans. Medical Imaging*, Dec. 1994

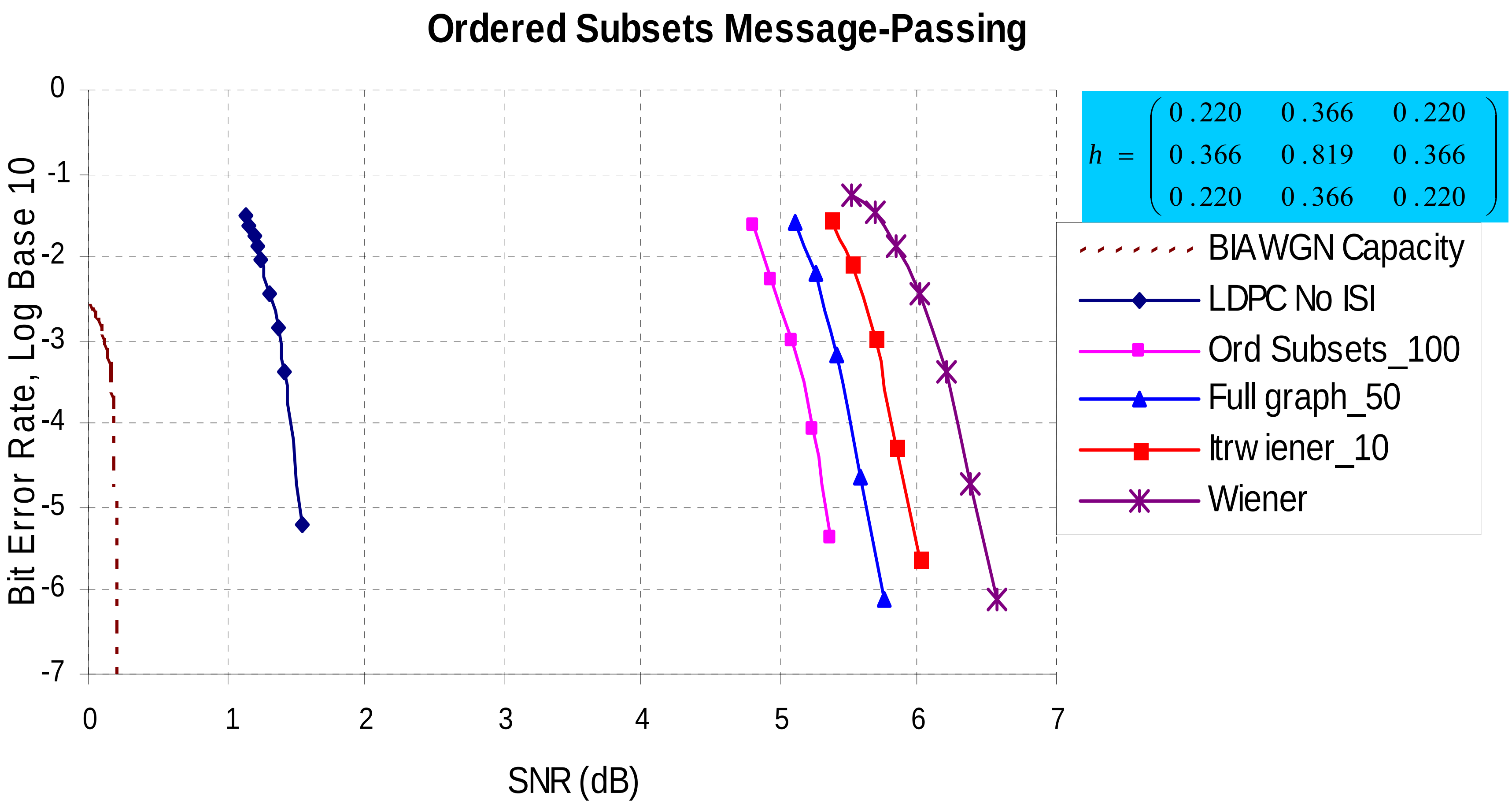
Grouped ISI Graph



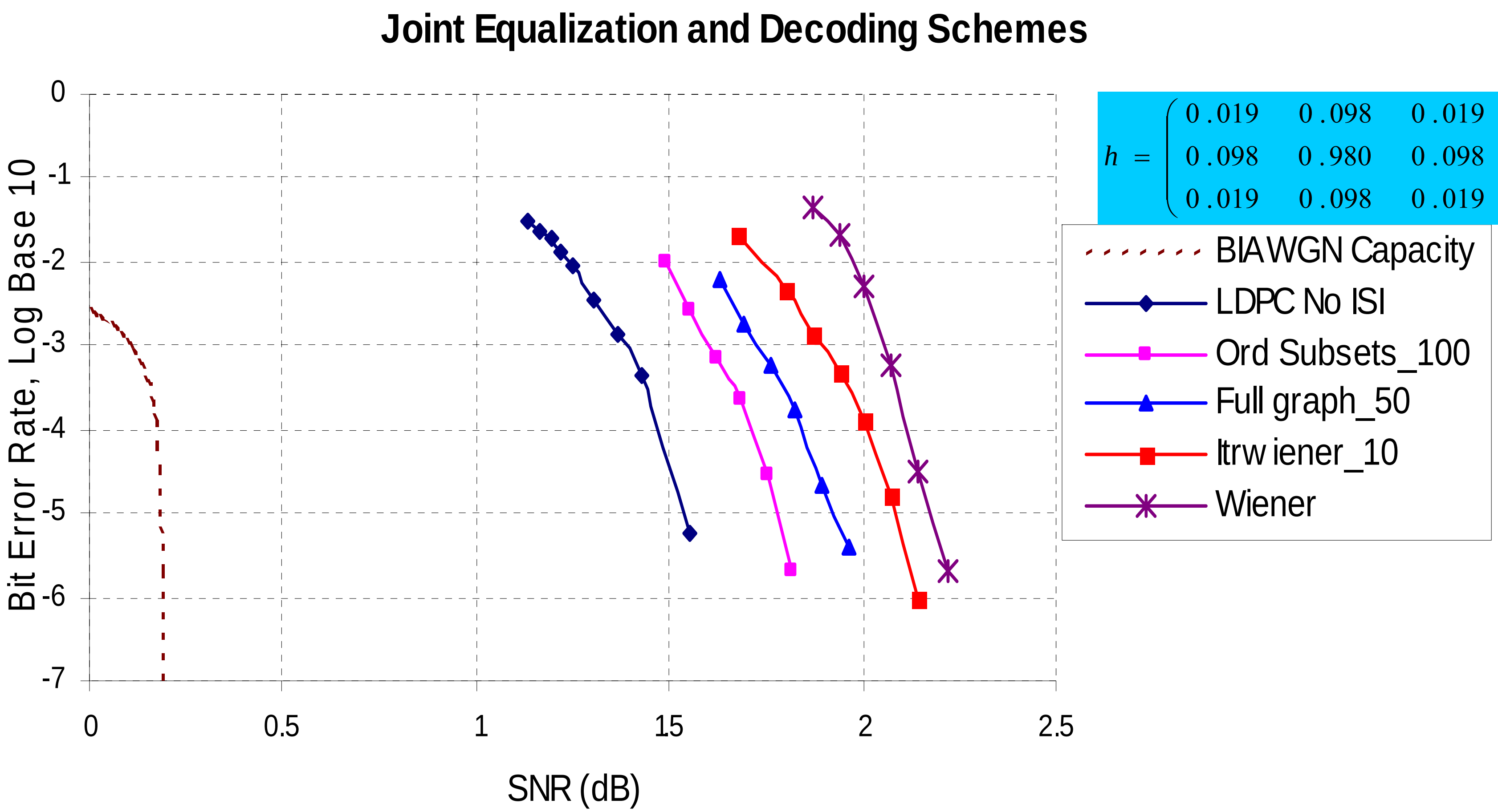
- Labeling of data nodes into 4 subsets
- For each iteration use data nodes of one label only

J. A. O'Sullivan, and N. Singla, "Ordered subsets message-passing," Submitted to *Int'l Symp. Inform. Theory* 2003.

Performance

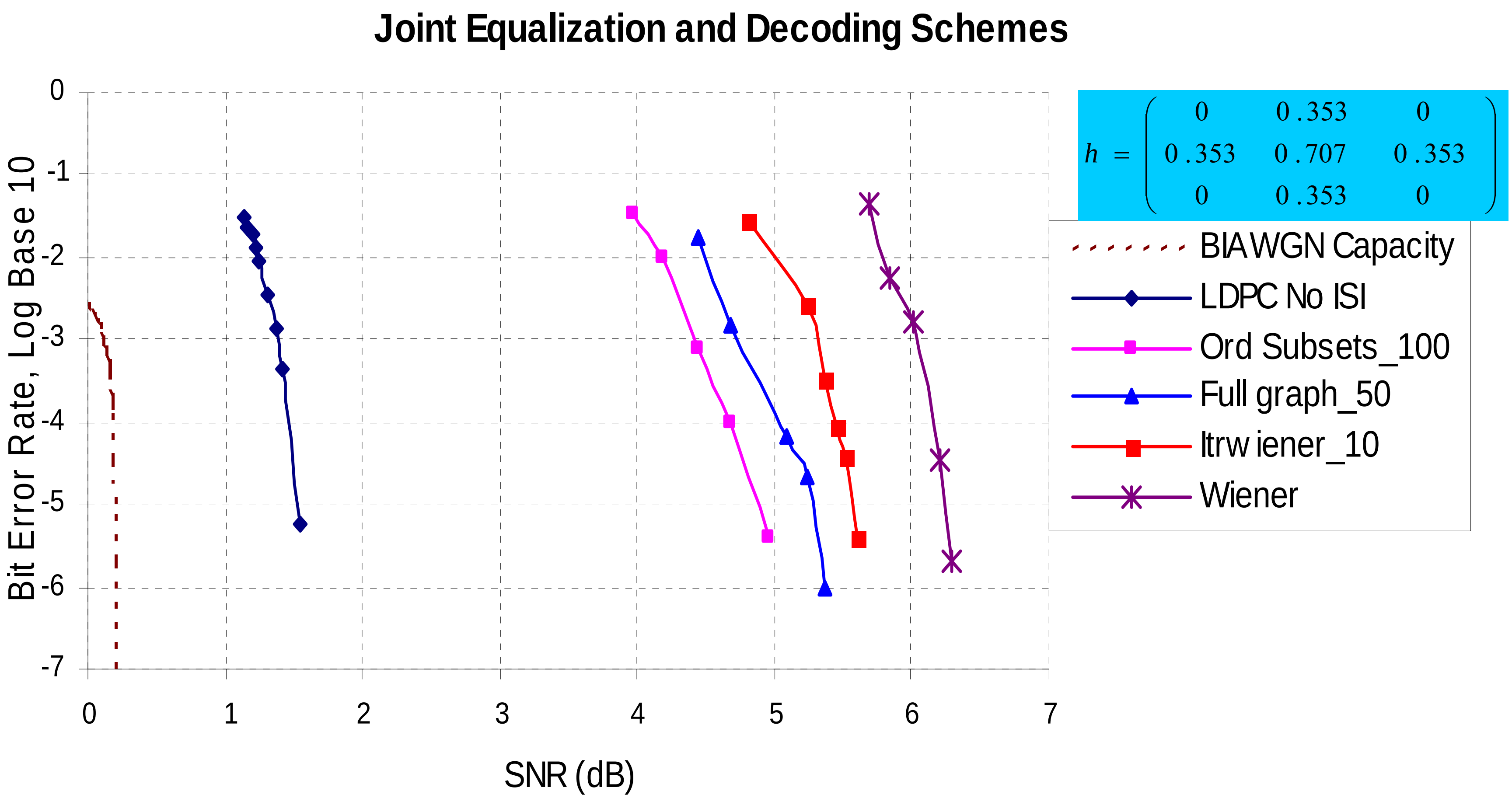


Performance

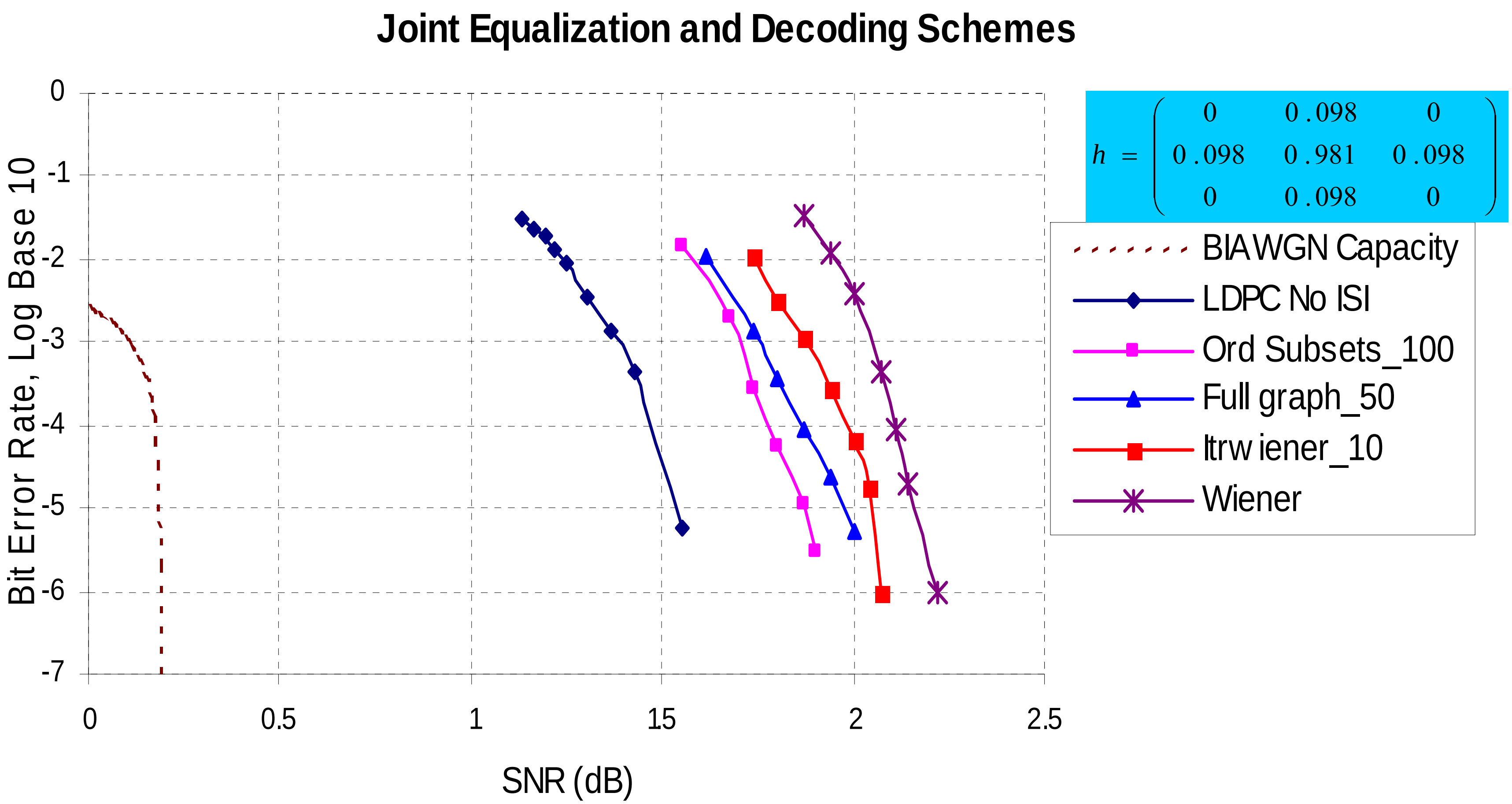


$$h = \begin{pmatrix} 0.019 & 0.098 & 0.019 \\ 0.098 & 0.980 & 0.098 \\ 0.019 & 0.098 & 0.019 \end{pmatrix}$$

Performance



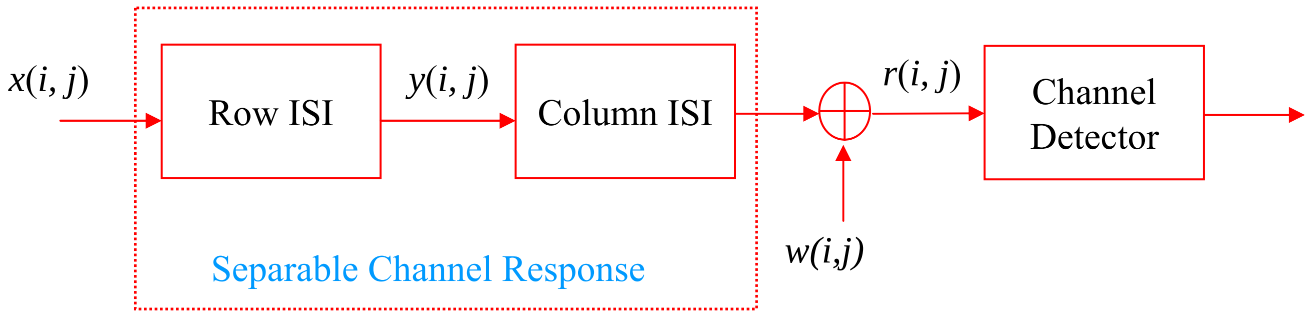
Performance



Consider Separable 2D ISI

Wu *et al.,* "Iterative detection and decoding for separable two-dimensional intersymbol interference" Submitted to *IEEE Trans. Magn.*, June. 2002.

A Separable 2D ISI

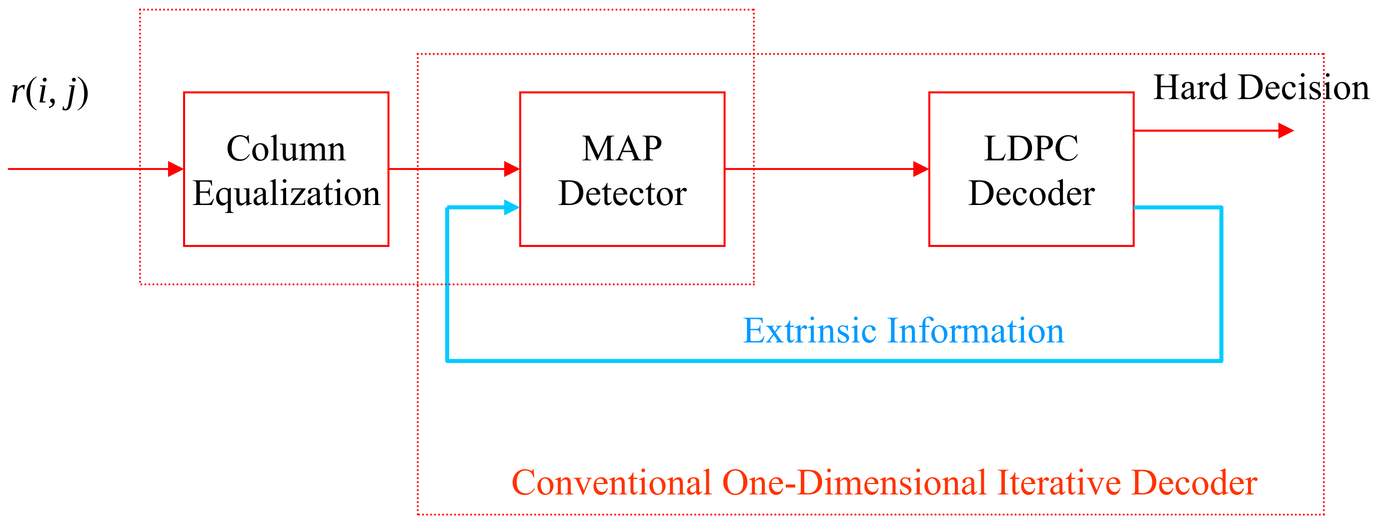


$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix} \begin{pmatrix} 1 & 0.5 \end{pmatrix}$$

- Advantages of Separable 2D ISI
  - ➢ Apply existing one-dimensional equalization methods
  - ➢ Reduced Detector Complexity

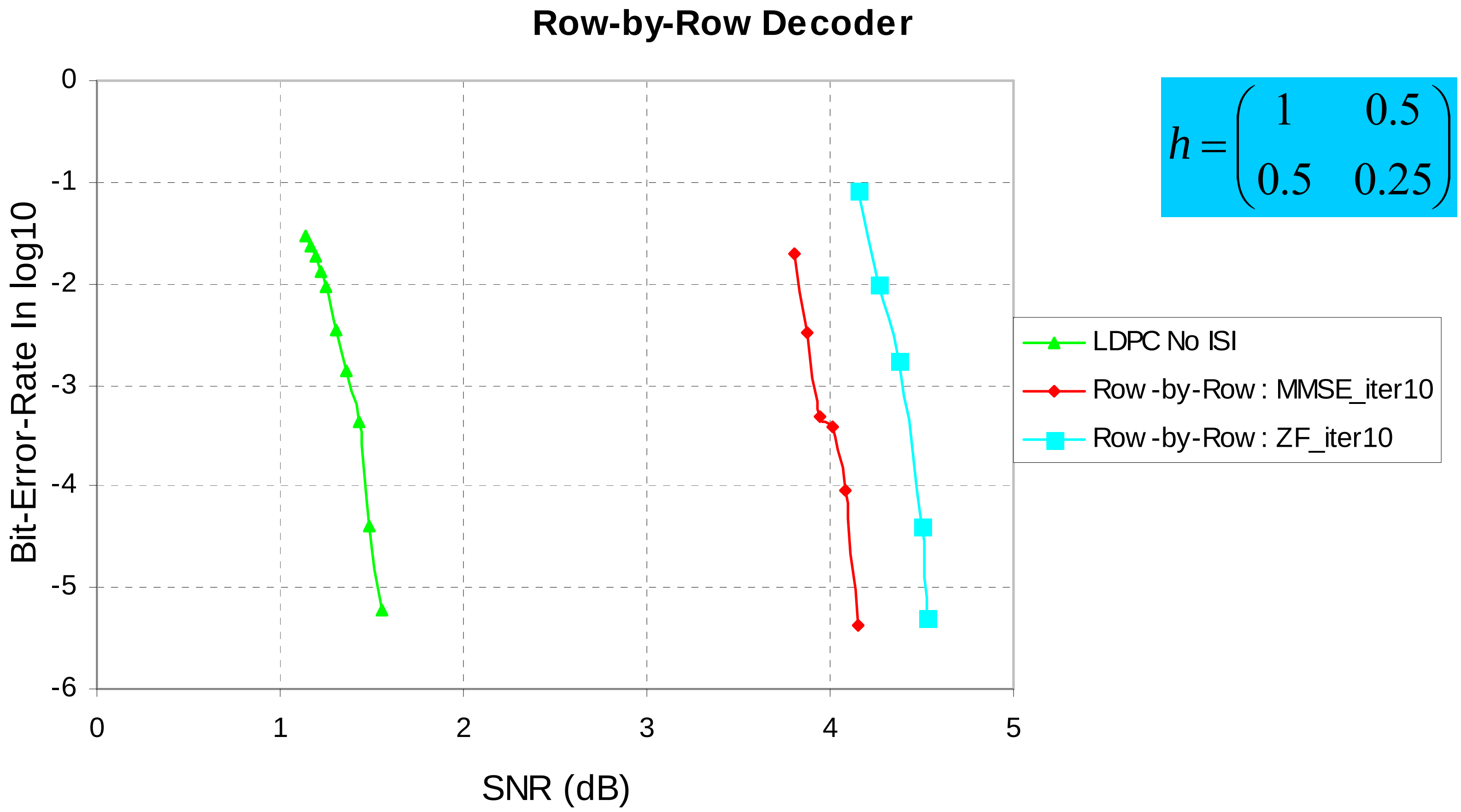
Tüchler *et al*., "Turbo equalization: principles and new results," *IEEE Trans. On Comm.*, May 2002.

Row-by-Row Decoder



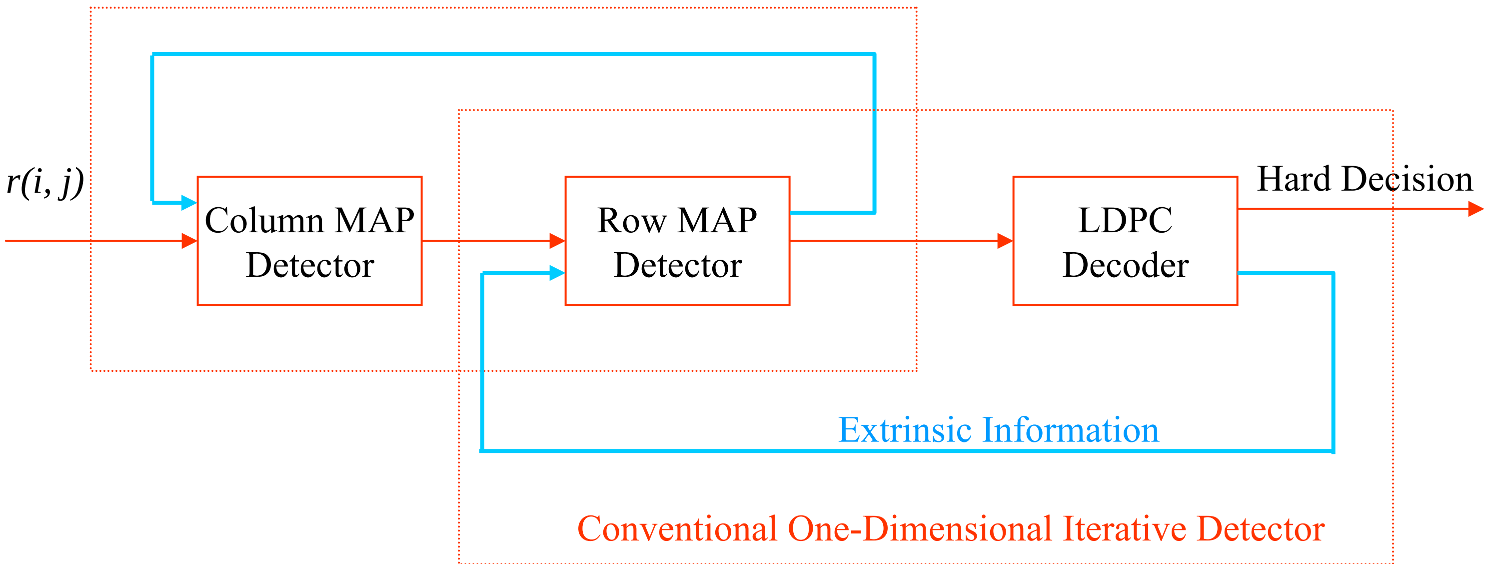
- MMSE and Zero-forcing criteria used for Equalization

Performance



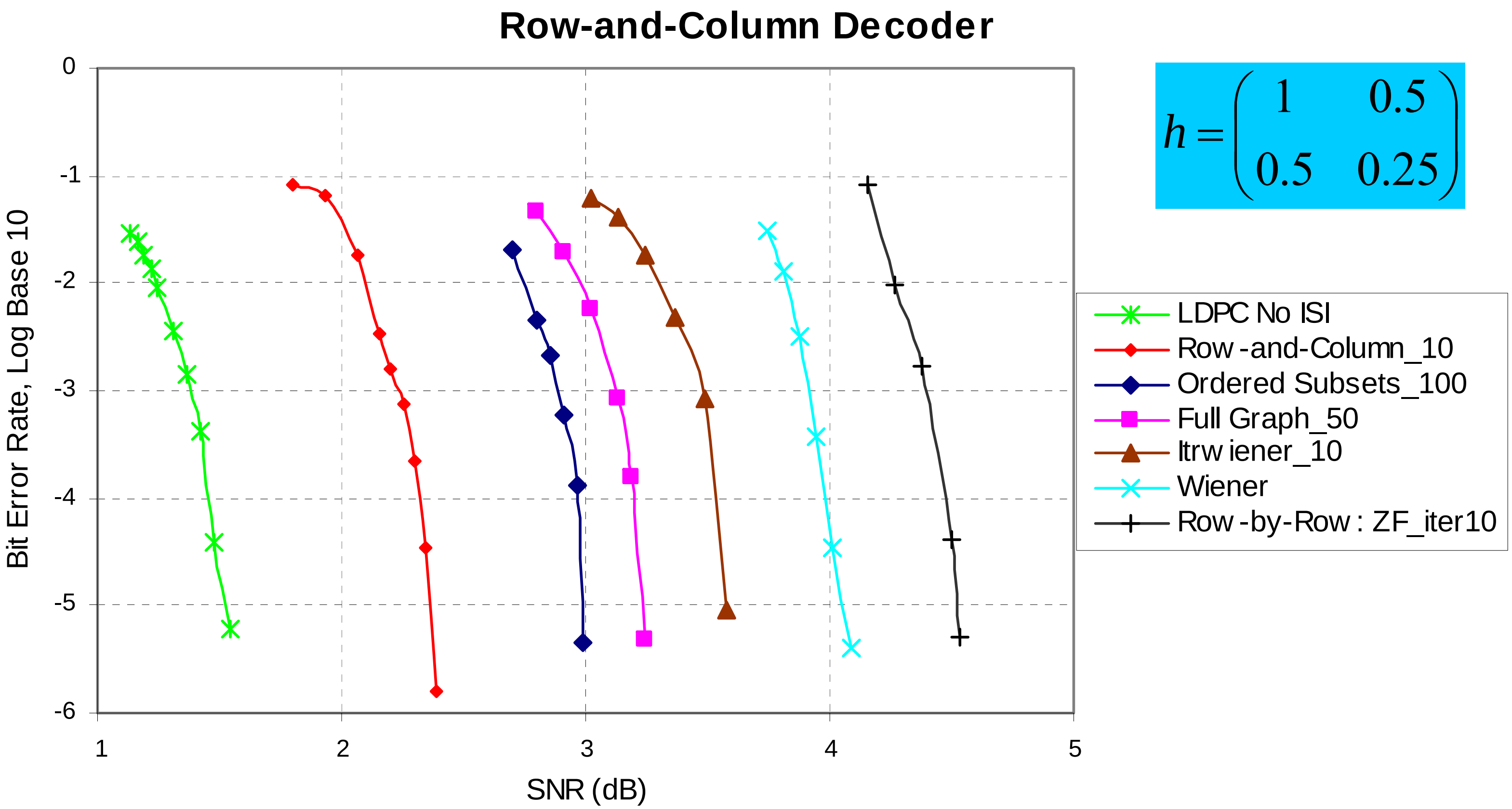
$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix}$$

Row-and-Column Decoder



- Inputs to column detector are not binary

Performance



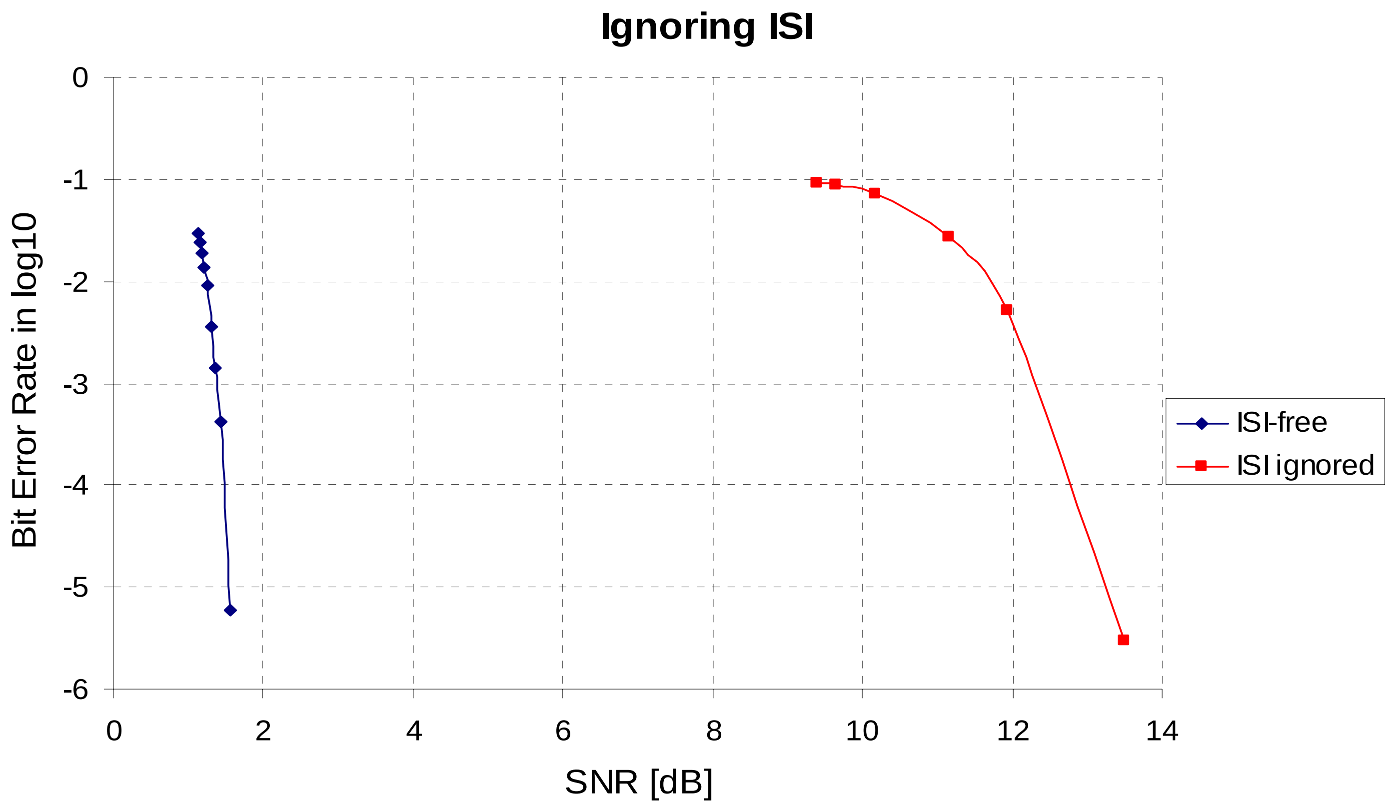
$$h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix}$$

Conclusions

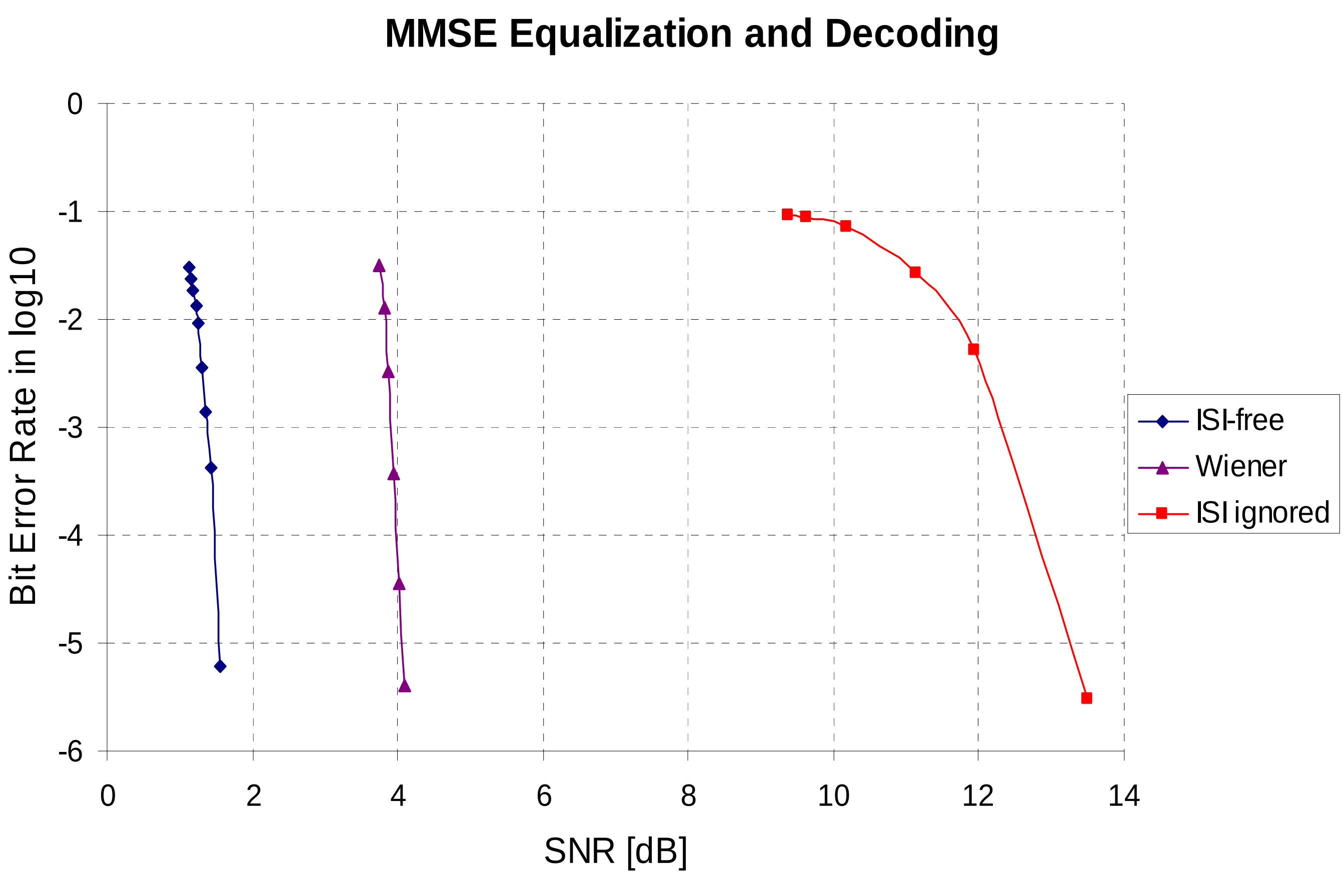
- MMSE equalization and decoding
  - Good Performance with Iterative Equalization
- Message-passing algorithms
  - Full graph algorithm performance deteriorated due to short cycles
  - Ordered subsets message-passing gives best performance for general 2D ISI
- Separable ISI decoding
  - Best performance for separable 2D ISI
  - Low complexity
  - Approximate channel response by separable response

Performance

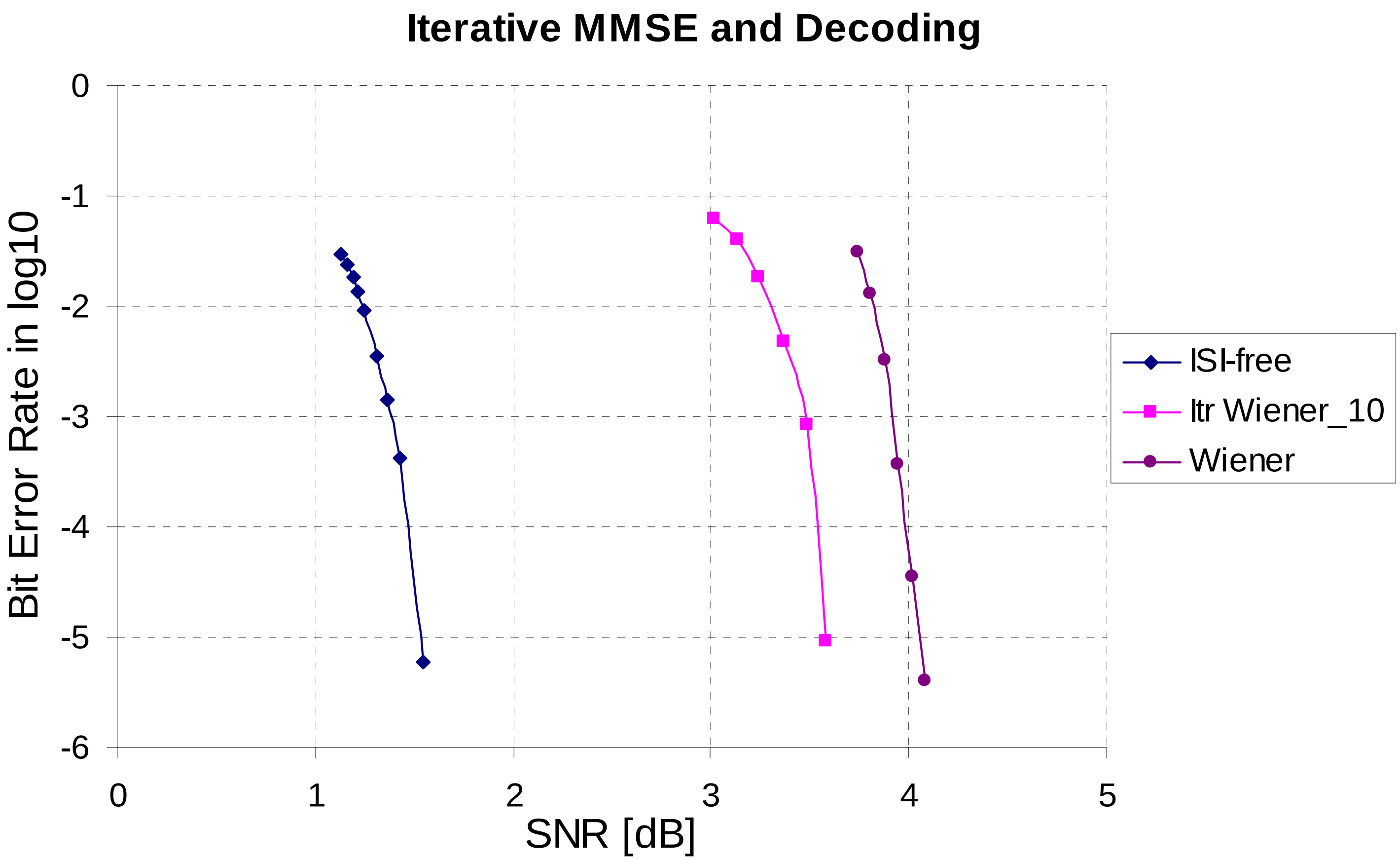
Block length 10000 regular (3,6) LDPC code



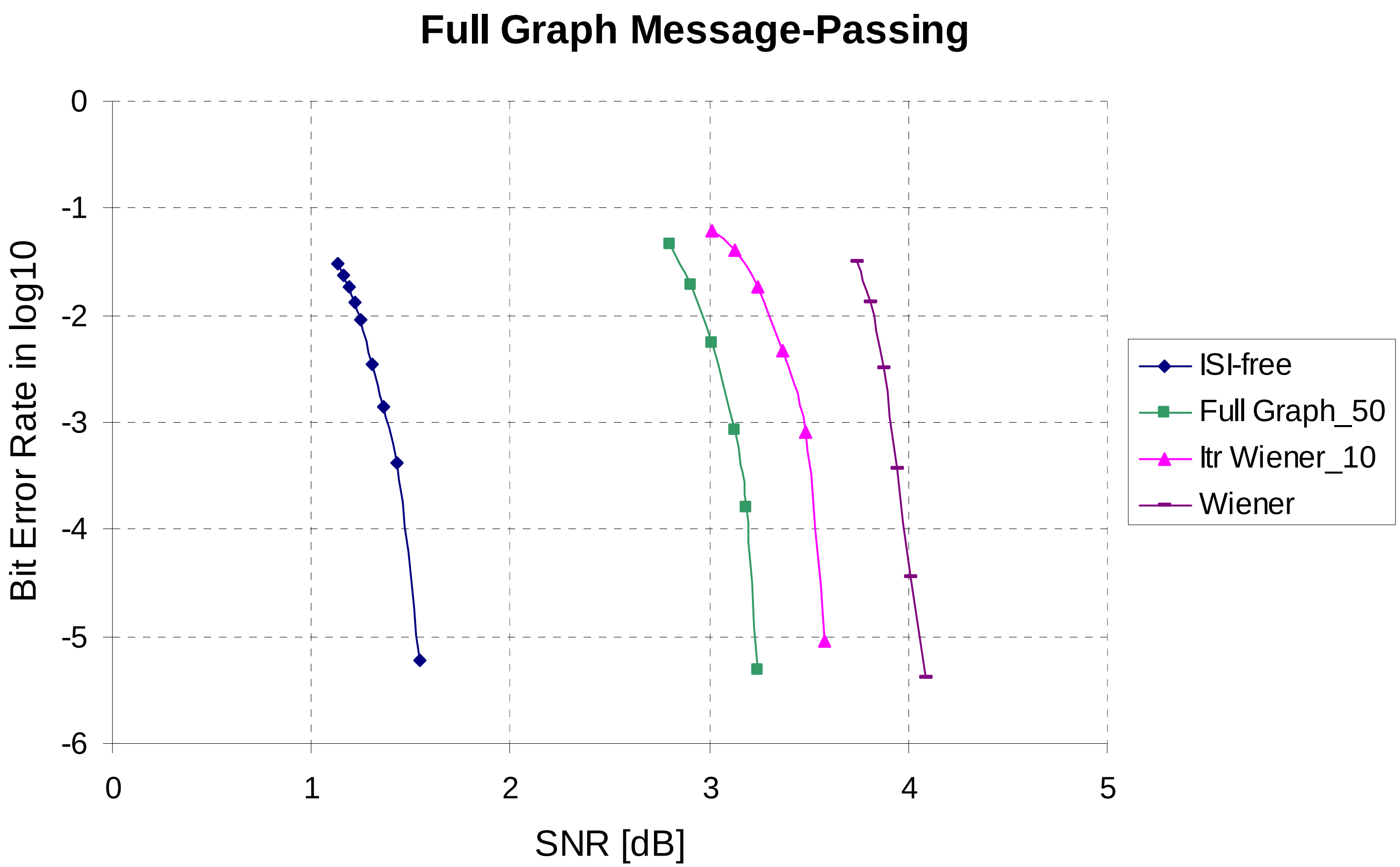
Performance



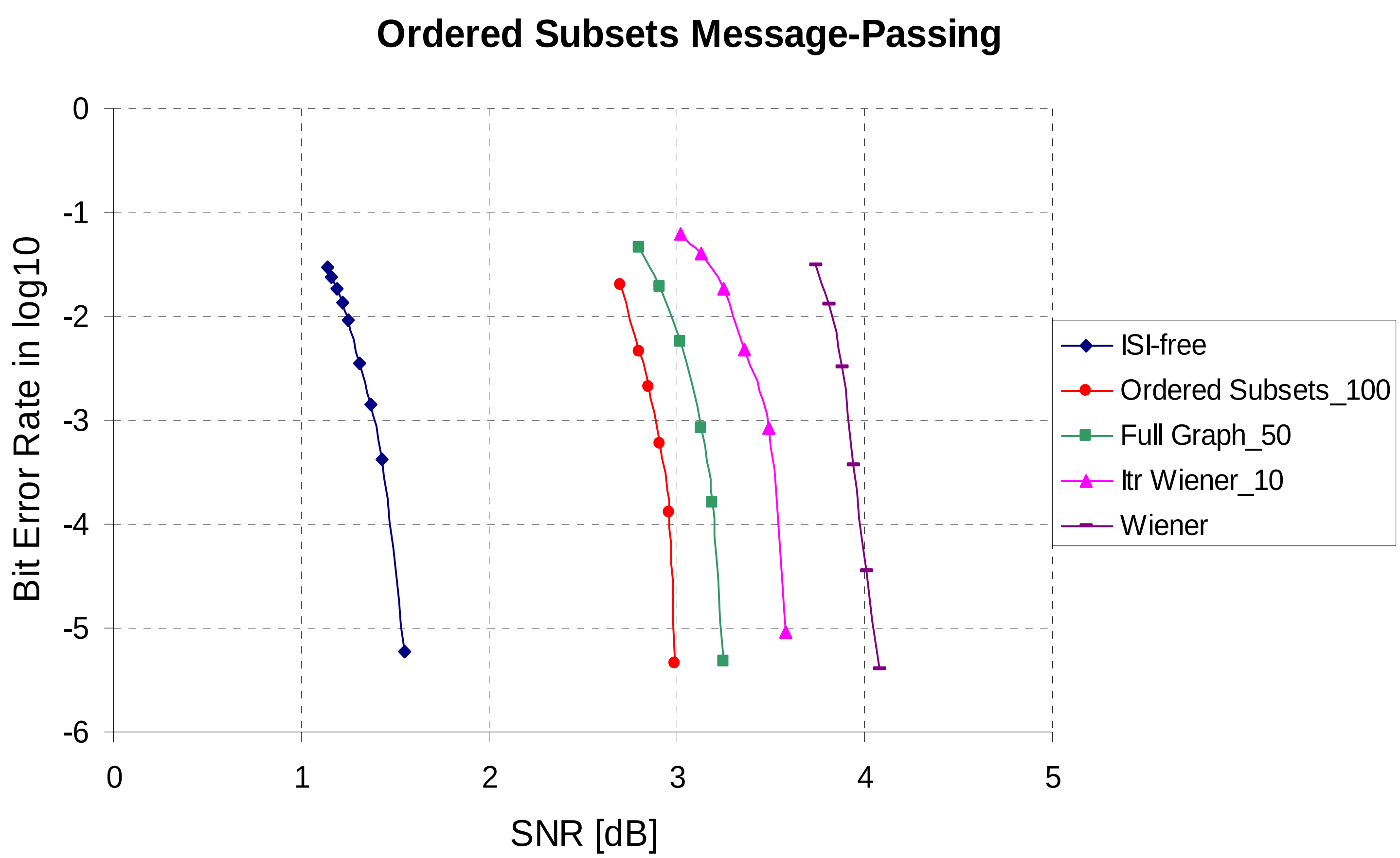
Performance



Performance



Performance



Advanced Media