Signal Processing for Advanced Storage Media

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Center for Security Technologies
Securing our World through Technology
Advanced Media for Storage

- Blu-ray disc
- Patterned media
- Holographic storage
- Two-dimensional optical storage

Two-Dimensional Intersymbol Interference

Existing schemes, like the Viterbi algorithm, cannot deal with 2D ISI due to complexity considerations.
Outline

- Joint Equalization and Decoding: Linear 2D ISI
  - MMSE Equalization
  - Full Graph Decoding
  - Modified Full Graph Decoding
  - Separable Channel Models
- 2D Optical Data Storage: Nonlinear ISI
  - Full Graph Decoding
  - Density Evolution for Threshold Behavior
- Conclusions
Joint Equalization and Decoding Schemes for 2D ISI

- Performance can be improved dramatically by combining error control coding with equalization
  - Based on existing equalization schemes
  - Jointly model channel ISI and parity check matrix for error control code—three level graph
  - Employ novel message-passing algorithms that take advantage of the 2D dependence

Low-density parity-check codes used for error correction

- $x(i,j) \in \{+1,-1\}$
- Channel ISI is 2D
- Noise is assumed to be AWGN
2D Linear Intersymbol Interference

\[
\begin{align*}
\begin{bmatrix}
1 & 1 & 1 & \Lambda & \Lambda & 1 \\
1 & x_{11} & x_{12} & \Lambda & x_{1k} & 1 \\
1 & x_{21} & \Omega & 1 \\
M & M & \Omega & \Omega & M \\
1 & x_{k1} & x_{k2} & \Lambda & x_{kk} & 1 \\
1 & 1 & 1 & \Lambda & \Lambda & 1
\end{bmatrix}
\end{align*}
\]

\[
h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix}
\]

\[
\begin{align*}
\begin{bmatrix}
r_{00} & r_{01} & \Lambda & \Lambda & \Lambda & r_{0k+1} \\
r_{10} & r_{11} & \Lambda & \Lambda & r_{1k} & r_{1k+1} \\
M & M & \Omega & M & r_{2k+1} \\
M & M & \Omega & M & M \\
r_{k0} & r_{k1} & \Lambda & \Lambda & r_{kk} & r_{kk+1} \\
r_{k+10} & r_{k+11} & \Lambda & \Lambda & r_{k+k} & r_{k+k+1}
\end{bmatrix}
\end{align*}
\]

Guard Band

\[
r_{i,j} = x_{i,j} + 0.5x_{i-1,j} + 0.5x_{i,j-1} + 0.25x_{i-1,j-1} + w_{i,j}
\]
MMSE Equalization

- Equalizer may or may not iterate with the LDPC decoder.
- Soft information, estimated mean of the codeword, passed from LDPC decoder to equalizer.
Performance

Block length 10000, regular, rate-0.5, LDPC code

Iterative MMSE and decoding

Bit error rate, log Base 10 vs. SNR [dB]

- ISI-free
- Itr Wiener_10
- Wiener
- MMSE-No coding

Block length 10000, regular, rate-0.5, LDPC code
Full Graph Message-Passing

\[ h = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{pmatrix} \]

\[ r_{i,j} = x_{i,j} + 0.5x_{i-1,j} + 0.5x_{i,j-1} + 0.25x_{i-1,j-1} + w_{i,j} \]
Full Graph Message-Passing

\[
L^{(l)}_{x \rightarrow z} = \sum_{m \in N(x)} L^{(l-1)}_{m \rightarrow x} + \sum_{z' \in N(x) \setminus z} L^{(l-1)}_{z' \rightarrow x}
\]
Full Graph Message-Passing

\[ \tanh \left( \frac{L_{z \rightarrow x}^{(l)}}{2} \right) = (-1)^z \prod_{x' \in N(z) \setminus x} \tanh \left( \frac{L_{x' \rightarrow z}^{(l-1)}}{2} \right) \]

- Check Nodes (z)
- Codeword Bit Nodes (x)
- Measured Data Nodes (r)
Full Graph Message-Passing

\[ L_{x \rightarrow m}^{(l)} = \sum_{m' \in N(x) \setminus m} L_{m' \rightarrow x}^{(l-1)} + \sum_{z \in N(x)} L_{z \rightarrow x}^{(l)} \]
Full Graph Message-Passing

Let \( L_{m \rightarrow x}^{(l)} = f(\{L_{x' \rightarrow m}^{(l)} : x' \in N(m) \setminus x\}) \)
Performance

Block length 10000, regular, rate-0.5, LDPC code

Full Graph Message Passing

Bit error rate in log10

SNR [dB]
Full Graph Analysis

- Length 4 cycles present which degrade performance of message-passing algorithm.

From Check Nodes:
- $x(i,j)$
- $x(i+1,j)$
- $x(i+2,j)$

To Check Nodes:
- \( \prod_{i,j} \)
- \( \prod_{i+1,j} \)
- \( \prod_{i+2,j} \)

Modified Full Graph Message-Passing

- From Imaging – Data set is grouped into subsets to increase rate of convergence
- For Decoding – Measured data is grouped into subsets and a modified schedule is employed: results in increase in girth of full graph

Labeling of data nodes into 4 subsets

For each iteration use data nodes of one label only

Performance

Block length 10000, regular, rate-0.5, LDPC code

Ordered Subsets Message Passing

Bit error rate in log10 vs. SNR [dB]
A Separable 2D ISI

- **Advantages of Separable 2D ISI**
  - Apply existing one-dimensional equalization methods
  - Reduced Detector Complexity

\[ h = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \begin{bmatrix} 1 & 0.5 \end{bmatrix} \]

Row-Column Decoder Diagram

- Inputs to column detector are not binary

Performance

Block length 10000, regular, rate-0.5, LDPC code

Row-Column Decoder

Bit error rate, log Base 10 vs. SNR [dB]
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Two-Dimensional Optical Storage

- 11 rows of hexagonal bit-cells stacked together
- Guard band separates adjacent stacks

2D ISI Model

\[ I(\mathbf{R}) = 1 - \sum_j c_j u_j + \sum_{j \neq k} d_{j,k} u_j u_k \quad j, k \in \mathbb{N}(\mathbf{R}) \]

- \( I(\mathbf{R}) \): received intensity at location \( \mathbf{R} \)
- \( c_j \): coefficients of linear ISI
- \( d_{j,k} \): coefficients of nonlinear ISI
- \( u_j \): binary data written on disc

- Based on scalar diffraction model proposed by Wim Coene
- Nonlinear ISI

2D ISI Model

- ISI coefficients calculated using recording specifics
- For simplicity use only nearest neighbors: 14 configurations
2D ISI: Signal Levels

Signal levels using nearest neighbors only

<table>
<thead>
<tr>
<th>Nonzero neighbors</th>
<th>Central bit=0</th>
<th>Central bit=1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.95</td>
<td>0.50</td>
</tr>
<tr>
<td>1</td>
<td>0.80</td>
<td>0.35</td>
</tr>
<tr>
<td>2</td>
<td>0.70</td>
<td>0.30</td>
</tr>
<tr>
<td>3</td>
<td>0.55</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>0.45</td>
<td>0.15</td>
</tr>
<tr>
<td>5</td>
<td>0.35</td>
<td>0.10</td>
</tr>
<tr>
<td>6</td>
<td>0.25</td>
<td>0.05</td>
</tr>
</tbody>
</table>

- Range of signal when central bit is 0 is greater than when central bit is 1: asymmetry due to nonlinear ISI

Full Graph Performance Results

Block length 10000, regular, rate-0.9, LDPC code
Density Evolution

- Assume messages are i.i.d. random variables
- Evolve message densities through the message maps
- If densities converge to desired density, then error-free transmission possible otherwise not
- Gives lower bound on performance of message-passing scheme

Density Evolution for Full Graph Message-Passing

- Codeword bit nodes to check nodes

\[ L^{(l)}_{x \rightarrow z} = \sum_{m \in N(x)} L^{(l-1)}_{m \rightarrow x} + \sum_{z' \in N(x) \setminus z} L^{(l-1)}_{z' \rightarrow x} \quad \text{CONVOLUTION} \]

- Check nodes to codeword bit nodes

\[ \tanh \frac{L^{(l)}_{z \rightarrow x}}{2} = (-1)^z \prod_{x' \in N(z) \setminus x} \tanh \frac{L^{(l-1)}_{x' \rightarrow z}}{2} \quad \text{LOOKUP TABLE} \]
Density Evolution…

- Codeword bit nodes to measured data nodes

\[ L^{(l)}_{x \rightarrow m} = \sum_{m' \in N(x) \setminus m} L^{(l-1)}_{m' \rightarrow x} + \sum_{z \in N(x)} L^{(l)}_{z \rightarrow x} \quad \text{CONVOLUTION} \]

- Measured data nodes to codeword bit nodes

\[ L^{(l)}_{m \rightarrow x} = f(\{L^{(l)}_{x' \rightarrow m} : x' \in N(m) \setminus x\}) \]

MONTE CARLO SIMULATION
# Density Evolution Results

Full graph algorithm for TWODOS

<table>
<thead>
<tr>
<th>Code Parameters $(d_v,d_c)$</th>
<th>Rate</th>
<th>Threshold Full Graph $(\sigma^2)$</th>
<th>SNR [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3,4)</td>
<td>0.25</td>
<td>0.0206</td>
<td>6.846</td>
</tr>
<tr>
<td>(3,6)</td>
<td>0.50</td>
<td>0.0071</td>
<td>8.462</td>
</tr>
<tr>
<td>(3,30)</td>
<td>0.90</td>
<td>0.0025</td>
<td>10.443</td>
</tr>
</tbody>
</table>
Complexity Considerations

- LDPC code complexity per iteration is linear in block length.
- At every iteration the number of computations on the channel ISI graph are proportional to number of edges in the channel ISI graph.
- Messages are floating point precision: fixed point implementation needed.
Conclusions

- Joint Decoding and Equalization
  - Prior simulations for magnetic media
  - Current simulations for optical hexagonal storage—account for nonlinearity
- Two-Dimensional Approach
- Potential SNR Improvement
Washington University Team

- **Professor Ronald S. Indeck**
  Expertise in magnetics, optics, experimental design, system integration
  - President of the IEEE Magnetics Society
  - Founder of Magnetics Information Systems Center at Washington University
  - Founding Director of the Center for Security Technologies at Washington University
  - Numerous national and international advisory appointments

- **Professor Joseph A. O’Sullivan**
  Expertise in information theory, imaging systems design and analysis, signal and image processing
  - Chair of the Washington University Faculty Senate
  - Director of the Electronic Systems and Signals Research Laboratory at Washington University
  - Associate Director of the Center for Security Technologies at Washington University
  - Past associate editor and publications editor for the IEEE Transactions on Information Theory

- Team of graduate students including Naveen Singla
- Synergistic research activities at Washington University include reconfigurable hardware for high speed computations