Iterative Decoding and Equalization for 2-D Recording

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Agenda

• Problem description
• LDPC codes
  – Introduction
  – Ensemble of codes
  – Decoding
  – Encoding
• Solutions and Results
• Future work
Motivation

• Why 2-D?
  – 1-D recording approaching saturation so move to 2-D to increase storage capacity.
  – LDPC and Turbo codes used as ECC codes for recording, work good for long lengths.

1-D, Sector is 4096 bits~1.3dB from capacity.
1024x1024=1048576~0.13dB from capacity.
Problem Description

- Read-back on recording channels suffers from ISI
- We consider 2-D recording media, 2-D ISI
- Use equalization techniques with LDPC code as ECC
- Why LDPC?
  - Random connection between coded bits
  - ISI is only local
LDPC Codes

- Linear block codes
- Completely defined by sparse $H_{m \times n}$
  $c$ is codeword iff $Hc = 0$
  Rate = $1 - \frac{m}{n}$
- Regular and Irregular codes
Bipartite Graph

- Bipartite graph representation
Ensemble of LDPC codes

- Degree Polynomials

\[ \lambda(x) = \sum_{i=1}^{d_v} \lambda_i x^{i-1} \]
\( \lambda_i \) : number of edges emanating from variable nodes of degree \( i \)

\[ \rho(x) = \sum_{i=1}^{d_c} \rho_i x^{i-1} \]
\( \rho_i \) : number of edges emanating from check nodes of degree \( i \)

\[ C^n(\lambda, \rho) \] Ensemble of length \( n \) codes
with \( (\lambda, \rho) \)
Decoding

- Maximum A Posteriori decoder

\( N(v) = \text{all } c \text{ s.t. } H(c,v) = 1 \text{ (1’s in a column)} \)

\( N(c) = \text{all } v \text{ s.t. } H(c,v) = 1 \text{ (1’s in a row)} \)

\( r = c + x : \text{ received word} \)

\( Hr = z = Hx : \text{ syndrome} \)

Calculate

\[
P(x \mid z) = \frac{P(x)P(z \mid x)}{P(z)}
\]
Message Passing

• Priors from channel output
• Check to variable messages

\[ r_{ml}^x = P(z_m \mid x_l = x) \]

\[ = \sum_{x_l': l' \in N(m) \setminus l; x_l = x} P(z_m \mid x_{l'}) P(x_{l'} \mid x_l = x) \]

\[ = \sum_{x_l': l' \in N(m) \setminus l} P(z_m \mid x_{l'}; x_l = 0) \prod_{l' \in N(m) \setminus l} q_{ml}^{x_{l'}} \]
Message Passing contd...

- Variable to check messages
  \[ q_{ml}^x = P(x_l = x | z_{m'}) = \frac{P(x_l = x)P(z_{m'} | x_l = x)}{P(z_{m'})} \quad m' \in N(l) \setminus m \]

- Posterior probabilities
  \[ q_{ml}^x = \alpha_{ml} p_l^0 \prod_{m'} r_{m'l} \]

- Complexity proportional to \( n \)

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Encoding

- Encoding is $O(n^2)$ – drawback
- Richardson and Urbanke, efficient encoder $O(n)$ for large enough block lengths
- Kavcic, LDPC coset codes

$$Hc = z; \ z_i \in \{0,1\}$$
Cycles

- Decoding algorithm converges to exact APP if done on a tree
- Our graph has cycles
  - Remove short cycles
- Richardson and Urbanke, performance concentrates around cycle free case with high probability as n gets large
Approach I

- $r = h^x + w$

$$h = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 0.25 \end{bmatrix}, \text{ } w \text{ AWGN}$$

- Assume $x$ is Gaussian and apply Wiener filter
Results I

[10000,5000] regular (3,5) code

EER

SNR(dB)
Approach II

- Iterate between Wiener filter and LDPC if single application fails
Results II

[10000,5000] regular (3,6) code

EER vs. SNR (dB)
More Results

[Graph showing data points and lines labeled 'capacity', 'normal', 'wiener', 'itwiener']
More Results contd…
Approach III

• Full graph decoder
\[ r_{ij} = h_{22} x_{i-1,j-1} + h_{21} x_{i-1,j} + h_{12} x_{ij-1} + h_{11} x_{ij} + w_{ij} \]
Full Message Passing
Full Graph contd…

- If LDPC fails then do full graph for fixed number of iterations.
- Scheduling: $v \rightarrow c \rightarrow v \rightarrow r \rightarrow v$. 
Results III
Comments

- Length 4 cycles present, degrade the performance.
- Use modification to ‘forget’ loops.
$$r_{ij} = h_{22} x_{i-1j-1} + h_{21} x_{i-1j} + h_{12} x_{ij-1} + h_{11} x_{ij} + w_{ij}$$
Comments

- Length 4 cycles present, degrade the performance.
- Use modification to ‘forget’ loops.
Future Work

- Irregular LDPC codes
- Capacity for binary input ISI channel
- Construct capacity approaching sequences